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Oscillations around tidal pseudo-synchronous solutions for circumbinary planets

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Abstract. Tidal evolution of low-eccentric circumbinary planets is expected to drive the rotational evolution toward a pseudo-synchronous solution. In this work, we present a study of the oscillation amplitudes around this state by considering that the two central stars exert creep tides on the planet. These amplitudes are computed by direct numerical integrations of the creep equations and also by means of the calculation of the coefficients of the periodic terms in this stationary solution. As in the two-body-problem, the planetary spin and lag-angle are observed to have maximum oscillation amplitudes for stiff bodies and almost null oscillation for the gaseous regime, while the opposite behaviour is observed in the equatorial and polar flattenings. Our analytical approximation shows to be very accurate and specially necessary for very-low eccentric planets. However, the magnitudes of the oscillation amplitudes around the pseudo-synchronous solution in the circumbinary problem appears to be very small respect to the mean value. Thus, considering these oscillation in the computation of the tidal energy dissipation may not have a substantial contribution in the results, at least compared to the case in which only the mean values are taken into account.

Keywords. planet-star interactions – methods: analytical

1. Introduction

One of the main characteristics shared by most circumbinary (CB) planets discovered by the *Kepler* mission is their location very close to the binary system. At such distances, tidal torques are expected to play an important role on the rotational evolution of the planet, with the peculiarity that in this context they are exerted by two central bodies of comparable mass, instead of just one.

In Zoppetti *et al.* (2019), we studied the rotational evolution of a CB planet due to the tidal interaction of the central binary, using the classical *Constant time-lag* model (Mignard 1979). Interestingly, we found that the typical stationary state of a CB planet is sub-synchronous respect to its mean motion, and this is exclusively due to the presence of the secondary star.

More recently, in Zoppetti *et al.* (2021) (hereafter Z2021) we investigate the effects of the binary tides on the CB planet with a more realistic formalism such as the creep tide model (Ferraz-Mello 2013). This formalism considers the bodies as Maxwellian fluids without the elastic component, and one of its main advantages is that the tidal-lags are not quantities *ad hoc* included in the theory, but are calculated from solving the set of differential equations derived from the Newtonian creep equation. Moreover, the theory does not need to assume weak friction, so this lag can be large and the model can be applied to bodies with arbitrary viscosities.

With this more general model, in Z2021 we could also find the sub-synchronous stationary solution for gaseous bodies, previously reported in Zoppetti *et al.* (2019). Furthermore, we provided a set of high-order analytical expressions for the mean values of the rotational stationary state around the 1:1 spin-orbit resonance. This last configuration, was shown to be like the most probable for low eccentric systems with viscosities in the range estimated for the planets of the Solar System (Ferraz-Mello 2013).

Although the mean values for the rotational stationary state obtained in Z2021 reproduce very well the behavior of the numerical simulations, we did not study in that article the oscillation of the real solution around these values. According to the 2-body-problem experience (e.g. Folonier et al. 2018), the oscillation amplitudes of the spin, for example, can become important for stiff bodies located very close to the perturber. In addition to the consequences to the potential habitability of these worlds, considering these amplitudes in the models is essential when estimating the dissipation of energy due to tides on the body and, also, the timescales and magnitudes of orbital evolution.

This article is organized as follows. In Section 2, we present the creep tide model applied to the case of the rotational evolution of a CB planet. In Section 3, we explain the method we used to obtain an analytical solution of the oscillation amplitudes around the pseudo-synchronous solution. Section 4 shows the results obtained for the case of Kepler-38 system. Finally, we discuss our main results and its implication in Section 5

2. The creep tide model for the rotational evolution of a CB planet

Let us consider an extended CB planet m_2 with radius $\mathscr{R}_2$, perturbed by the tidal interaction of a central binary with components m_0 and m_1. Additionally, let us consider the planar problem where the spin vector of the CB planet is perpendicular to the orbital plane.

As a consequence of the gravitational interaction exerted by the central stars, the CB planet undergoes a tidal deformation. In addition to this, in this work we also take into account the rotational flattening on m_2 due to its own spin, and assume that the resulting deformation due to these effects is small enough that a model developed up to the first order of the flattenings represents an accurate description of the problem.

In the creep tide theory (Ferraz-Mello 2013), the real shape and orientation of the planet is computed by means of the distance ζ_2 (measured from the center of mass of the body) of an arbitrary point on its surface with co-latitude ϑ and longitude φ. Its explicit form is given by

$$\zeta_2(\vartheta,\varphi) = \mathscr{R}_2\left[1 + \mathscr{E}_2^z\left(\frac{1}{3} - \cos^2\vartheta\right) + \frac{\mathscr{E}_2^\rho}{2}\sin^2\vartheta\cos\left(2\varphi - 2(\varphi^{eq} + \delta_2)\right)\right], \tag{1}$$

where $\mathscr{E}_2^r$ and $\mathscr{E}_2^z$ are the equatorial and tidal flattenings, δ_2 is the lag angle and φ^{eq} is the position of a fictitious body that exerts a tide on the planet equivalent to that exerted by the two central stars (see Section 2.1 of Z2021)

The parameters that characterize the shape ($\mathscr{E}_2^\rho$ and $\mathscr{E}_2^z$) and orientation (δ_2) of the planet, can be calculated by solving the creep tide equation

$$\dot{\zeta}_2 + \gamma_2\zeta_2 = \gamma_2\rho_2, \tag{2}$$

where γ_2 is the relaxation factor of the planet, which is inversely proportional to its viscosity and assumed as constant in this work, while ρ_2 is the surface equation of the

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equilibrium figure (see Equation (3) of Z2021). The explicit set of differential equations for the shape and orientation of the planet is

$$\begin{aligned} \dot{\delta}_2 &= \Omega_2 - \varphi^{eq} - \frac{\gamma_2 \varepsilon_2^{\rho}}{2\mathscr{E}_2^{\rho}} \sin(2\delta_2) \\ \dot{\mathscr{E}}_2^{\rho} &= \gamma_2(\varepsilon_2^{\rho} \cos(2\delta_2) - \mathscr{E}_2^{\rho}) \\ \dot{\mathscr{E}}_2^{z} &= \gamma_2(\varepsilon_2^{z} - \mathscr{E}_2^{z}), \end{aligned} \tag{3}$$

where ε_2^{ρ} and ε_2^{z} are the equivalent equatorial and polar flattenings of the equilibrium figure (for its explicit expressions, see equations (2) and (4) of Z2021).

For a given planetary spin Ω_2, the real shape and orientation of the body can be calculated by solving the differential equation system (3). Then, the variation in the planetary rotation is derived from the reaction torques that the extended body feels and the conservation of total angular momentum. Neglecting the term corresponding to the variation of the polar moment of inertia, its explicit form is

$$\dot{\Omega}_2 = -\frac{2\mathscr{G} m_2}{5\mathscr{R}_2^3} \varepsilon_2^{\rho} \mathscr{E}_2^{\rho} \sin(2\delta_2). \tag{4}$$

The set of differential equations (3) and (4) describe the rotational evolution of the CB planet due to the creep tides of the central binary.

In Z2021, we consider planetary relaxation factors in the range estimated for the Solar System planets, and found that the most probable rotational stationary state is the pseudo-synchronous solution, at least for low eccentric CB systems. For this reason, the oscillation amplitudes around this particular solution will be the target of this work.

On the other hand, in the framework of the *Constant-time-lag* model, in Zoppetti *et al.* (2019) and Zoppetti et al. (2020) we observed that the characteristic timescales of the rotational evolution of *Kepler* CB planets are typically much shorter than the timescales of the orbital evolution. For this reason, in this work we also take advantage of this adiabatic nature of the problem and solve the set of differential equations given by (3) and (4), assuming fixed values for all the orbital elements except the mean anomalies.

3. Analytical resolution method for the pseudo-synchronous stationary state

We adopt here the same procedure adopted in Z2021. We then choose a Jacobi reference frame and propose a particular stationary solution inspired in the functional dependence of the elliptical expansions of ε_2^{ρ}, ε_2^{z} and φ^{eq} of the form

$$\begin{aligned} \Omega_2 &= \sum_{\vec{\ell}} \{\Omega_2\}_{\vec{\ell}} \cos\left(l_1 M_1 + l_2 M_2 + l_3 \varpi_1 + l_4 \varpi_2 - \Phi_{\vec{\ell},\Omega_2}\right) \\ \mathscr{E}_2^{\rho} &= \sum_{\vec{\ell}} \{\mathscr{E}_2^{\rho}\}_{\vec{\ell}} \cos\left(l_1 M_1 + l_2 M_2 + l_3 \varpi_1 + l_4 \varpi_2 - \Phi_{\vec{\ell},\mathscr{E}_2^{\rho}}\right) \\ \delta_2 &= \sum_{\vec{\ell}} \{\delta_2\}_{\vec{\ell}} \cos\left(l_1 M_1 + l_2 M_2 + l_3 \varpi_1 + l_4 \varpi_2 - \Phi_{\vec{\ell},\delta_2}\right) \\ \mathscr{E}_2^{z} &= \sum_{\vec{\ell}} \{\mathscr{E}_2^{z}\}_{\vec{\ell}} \cos\left(l_1 M_1 + l_2 M_2 + l_3 \varpi_1 + l_4 \varpi_2 - \Phi_{\vec{\ell},\mathscr{E}_2^{z}}\right), \end{aligned} \tag{5}$$

where M_1 and ϖ_1 are the mean anomaly and pericentre longitude of the secondary star, while M_2 and ϖ_2 are those corresponding to the planet. We note that each of the rotational evolution variables $w = \Omega_2, \mathscr{E}_2^{\rho}, \delta_2$ or $\mathscr{E}_2^{z}$, the amplitudes $\{w\}_{\vec{\ell}}$ and the constant phases $\Phi_{\vec{\ell},w}$ depend on the subscripts $\vec{\ell} = (l_1, l_2, l_3, l_4)$.

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Table 1. Initial conditions of our Kepler-38 like system (Orosz et al. 2012). Orbital elements are given in a Jacobi reference frame.

body	m_0	m_1	m_2
mass	$0.95\, m_\odot$	$0.25\, m_\odot$	$10\, m_\oplus$
radius	$0.84\, \mathscr{R}_\odot$	$0.27\, \mathscr{R}_\odot$	$4.35\, \mathscr{R}_\oplus$
a_i [AU]		0.15	0.45
e_i		0.15	$0.0 - 0.1$

The particular solution given in (5) and its derivatives are then introduced into the system (3) and (4). Having expanded the forced terms (ε_2^ρ, ε_2^z and φ^{eq}) in power series of semimajor axis ratio $\alpha = a_1/a_2$ and the eccentricities e_1 and e_2, the coefficients can be explicitly calculated by equating the terms with the same trigonometric argument and neglecting terms of higher order.

The terms in system (5) with $l_1 = l_2 = 0$ correspond to the mean values of the rotational stationary solution and were explicitly given in Equation (16)-(19) of Z2021.

In this work, we focus on the amplitudes of the periodic terms. The calculation of these coefficients is even more cumbersome than in the case of the mean values. For this reason, we computed them up to 3rd order in α and only up to 1st-order in the eccentricities. However, as we will see in Section(4), we are able to predict very well the oscillation amplitudes of low-eccentric CB planets close to their host binary. On the other hand, due to space limitations, the coefficients can not be explicitly shown in this manuscript but can be provided by contacting any of the authors.

4. Results

We apply the results of our model to a Kepler-38-like CB planet (Orosz et al. 2012). This system has been the test case of most of our recent investigations about the tides on CB planets. The chosen nominal values for system parameters and initial orbital elements are detailed in Table 1. The value of m_2 was estimated from a semi-empirical mass–radius fit (Mills & Mazeh 2017). Note that two different values of the planetary eccentricity e_2 are considered in this work: $e_2 = 0.0$ and $e_2 = 0.1$

In Figure 1, we consider a Kepler-38-like CB planet and show the oscillation amplitudes, predicted by our creep tide model, for the rotational evolution around the pseudo-synchronous solution, as a function of the normalized relaxation factor γ_2/n_2. The wide black lines correspond to the amplitudes obtained by numerically integrating the system (3) and (4), once the planet reaches the stationary solution, while the dashed black curves correspond to the mean values obtained in this integration. The red curves represent the results of our analytical solution up to 3rd-order in α, obtained by means of the method described in Section 3, while the results of our 0th-order analytical solution is represented by green curves. Note that this last case is equivalent to the 2-body-problem in which the planet only feels the tides of one central body with mass equal to the sum of both stellar masses.

We first note from Figure 1 that, independently of the planetary eccentricity, the oscillation around the pseudo-synchronous planetary spin shows the same behavior than the one around the mean lag-angle: it is maximum for $\gamma_2 << n_2$ (i.e. typically stiff bodies) and decays to zero for $\gamma_2 >> n_2$ (i.e. gaseous bodies). On the other hand, the completely opposite behavior is observed for the flattenings $\mathscr{E}_2^\rho$ and $\mathscr{E}_2^z$. The magnitudes of the oscillation amplitudes do depend (proportionally) on the planetary eccentricity, although in our case they are much smaller than the ones obtained for the 2-body-problem Saturn-Enceladus in Folonier et al. (2018).

Regarding to the accuracy of our analytical solution, we note from Figure 1 that our model up to 3rd-order in α fits very well the behaviour of the numerical integrations,

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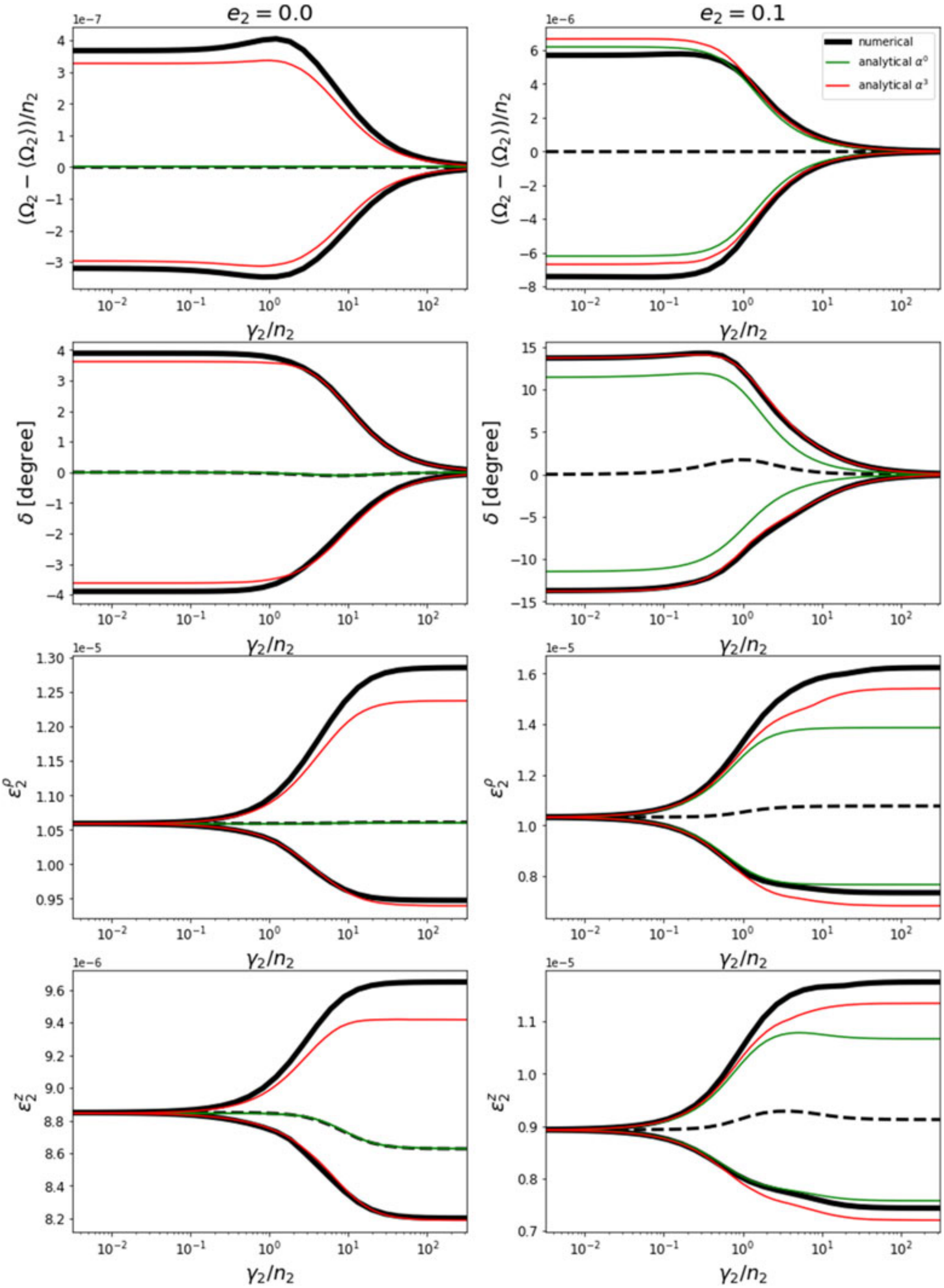

Figure 1. Oscillation amplitudes of the spin Ω_2 (first row), the lag-angle δ_2 (second row), the equatorial flattening ε_2^ρ (third row) and polar flattening ε_2^ρ (bottom row), of a Kepler-38-like planet around the pseudo-synchronous solution. Different colors indicate different methods for obtaining the amplitudes: numerical in black, analytical up to 0th-order in green and analytical up to 3th-order in red. The dashed black lines represent the numerical mean solution. Different columns represent different planetary eccentricities: $e_2 = 0.0$ (left column) and $e_2 = 0.1$ (right column).

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for any arbitrary planetary viscosity in low eccentric orbits. The accuracy of the model is even more remarkable (for example, see the second row panels) when we take into account that in CB environments, tides are non-negligible for planets very close to the binary and high α-order expansions are required for the analytical models (Figure 1 is built for $\alpha = 0.33$).

On the other hand, the comparison between the red and the green curves for the panels located on the left columns, shows that the 3BP approximation is specially necessary to compute the oscillation amplitudes of very-low-eccentric orbits, where the 2BP approach predicts null oscillations (see Equations (53) of Folonier et al. (2018)) and the numerical solution is far from fulfilling it. However, the 0th-order solution seems to be quite acceptable for moderate eccentricities.

5. Discussion

In this work, we study the oscillation amplitudes that exhibits the rotational evolution of a CB planet around the pseudo-synchronous tidal solution. We employ the creep tide model (Ferraz-Mello 2013), which is equivalent to considering bodies as Maxwellian fluids without the elastic component (e.g. Ferraz-Mello 2015).

We compute the oscillation amplitudes in two ways: by a direct numerical integration of the differential equation system of the rotational evolution, and by analytically calculating the coefficients of the periodic terms of the stationary pseudo-synchronous state. As discussed in Z2021, having analytical expressions is a very cumbersome task. However, it allows to carry out semi-analytical studies of the orbital evolution for very long periods of time, by introducing *ad-hoc* the solutions found for the rotational evolution, and avoiding having to solve our entire multi-timescales system. The analytical approach presented in this work fits very well the numerical predictions of low eccentric systems, even with high semimajor axis ratio, so it can be an important contribution in this problem.

For CB planets, the behaviour observed in the oscillation amplitudes is analogous to the one observed in the 2-body-problem: maximum oscillation amplitudes for the spin and lag-angles of stiff planets and almost zero amplitudes in the gaseous regime, while the completely opposite behaviour is observed for the flattenings. However, the magnitudes of the oscillation observed for our Kepler-38-system (whose parameters and orbital elements are typical within the planets observed by the *Kepler* mission) are very small, specially when we compare with the ones obtained for the Saturn-Enceladus system (Folonier et al. 2018), this last being a much tighter system. The presence of an inner instability limit for CB orbits (e.g. Holman & Wiegert 1999), prohibits to have planets much closer to the binary than Kepler-38. Thus, the expected oscillation amplitudes for the parameters that characterise the rotational evolution of CB planets are expected to be typically very small.

We mention two important consequences of this result. On one hand, the very low oscillation amplitudes of the spin of CB planets that have reached the pseudo-synchronous solution should be taken into account in habitability studies of these planets. On the other hand, from a technical point of view, in this type of environment, building models that consider only the mean behavior and neglect the oscillation amplitudes seems to be very accurate.

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