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# The formation of diffusive staircases

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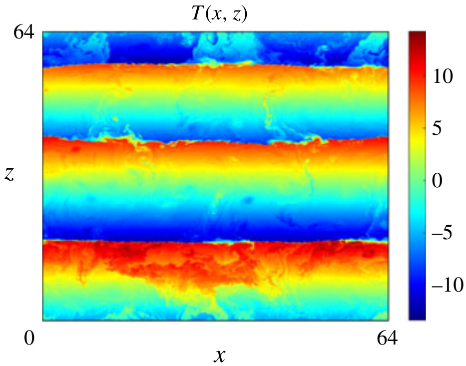


Radko's recent article (*J. Fluid Mech.*, vol. 805, 2016, pp. 147–170) entitled Thermohaline layering in dynamically and diffusively stable shear flows, is slated to become a seminal reference in the field of fluid dynamics. It proposes an elegant solution to the long-standing question of why thermohaline staircases form in the high-latitude oceans. Equally importantly, it provides a rare and interesting example of how two physical processes that are both strongly stable when considered individually, can trigger a linear instability when they interact.

**Key words:** convection, double-diffusive convection

## 1. Introduction

One of the main foci of Radko's research in the past 20 years has been the study of double-diffusive convection and thermohaline staircases. Double-diffusive convection refers to a class of buoyancy-driven instabilities that occur when density depends on two scalar fields that diffuse at different rates, such as fingering convection (Stern 1960) and oscillatory double-diffusive convection (Walin 1964). Meanwhile, thermohaline staircases are usually found in stratified and relatively quiescent water masses, including the tropical and subtropical oceanic thermocline (e.g. Schmitt *et al.* 1987), in the high-latitude oceans (e.g. Timmermans *et al.* 2008) and in some geothermal lakes (e.g. Schmid, Busbridge & Wüest 2010). As their name suggests, they exist in both thermally and salt-stratified fluids, and present themselves as stacks of horizontal well-mixed layers separated by thin interfaces. These staircases can be classified into two groups: those for which temperature and salinity both decrease with depth (as in the case of tropical and subtropical staircases), called fingering staircases hereafter, and those for which they both increase with depth (as in the polar oceans and geothermally heated lakes), called diffusive staircases. Almost all naturally occurring staircases of either type are found in regions that

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are double-diffusively unstable, either through linear or finite amplitude instabilities, suggesting a strong connection between the two phenomena.

The existence of thermohaline staircases has been known for a long time, and theories for their formation abound. Some rely on double-diffusive processes, such as intrusive interleaving in the presence of horizontal temperature and salinity gradients (Holyer 1983), or the breaking of internal gravity waves excited by a collective instability of double-diffusive fingering (Stern & Turner 1969). Others rely on anti-diffusive mixing caused by mechanical stirring (Balmforth, Llewellyn-Smith & Young 1998). While most of these mechanisms likely participate in the formation of staircases somewhere in the world, the key question is what the fastest and most robust one is for each of the two main classes of staircases described above. In 2003, Radko definitely solved the problem for fingering staircases, showing that they arise from a mean-field instability of fingering convection, the $\gamma$-instability (Radko 2003). The $\gamma$-instability theory has turned out to be easily generalizable to other systems, and has been used to model the formation of diffusive staircases in stars and planets (Mirouh *et al.* 2012, and related publications) for instance. One could therefore naively expect that it would also explain the formation of diffusive staircases in lakes and in the polar oceans.

As pointed out by Radko, however, the mean diffusive density ratio (the ratio of the stabilizing density gradient due to salt to the destabilizing density gradient due to temperature) of these staircases ($R_\rho \sim 2$–$10$) is significantly larger than the critical value for instability to oscillatory double-diffusive convection, which is approximately 1.1 for salt water, placing them deeply into the stable region of parameter space. Without a basic instability to build upon, the $\gamma$-instability theory must be abandoned and alternatives sought. In this paper, Radko proposes a new mechanism for instability in strongly stratified double-diffusive systems that could well solve the long-standing diffusive staircase formation problem.

## 2. Layer formation in double-diffusive shear instabilities

At the heart of Radko's work lies a simple yet revolutionary discovery: that a doubly stratified fluid far within the stable region of parameter space for oscillatory double-diffusive convection can be destabilized by a small amount of shear. Small in this context implies that the shear has a gradient Richardson number $Ri$ (the ratio of the square of the local buoyancy frequency to the square of the local shearing rate) that is everywhere much greater than one, thus lying well within the stable region of parameter space for stratified shear instabilities. In other words, Radko has found that these two individually stable processes, when combined, give rise to a new linear instability, now called the thermohaline shear instability.

Shear is always present in the ocean, driven by various possible sources ranging from basin-scale currents to the internal wave field. Focussing on the simplified case of temporally steady but spatially periodic shear, Radko uses linear theory to show that this new instability can exist for $R_\rho$ as large as 50 even when $Ri \sim O(100)$. The fastest growing mode of instability has a vertical wavelength commensurate with that of the shear. Its growth time scale depends strongly on all system parameters, but seems to be of the order of a few months for reasonable estimates of flow velocities and stratification in the high-latitude oceans.

The unexpected appearance of a linearly unstable mode from the combination of two stable processes can be understood as follows. The shear distorts the vertical double-diffusive mode, causing it to buckle. The effect is more pronounced for salinity than for temperature since the latter diffuses faster. The temperature and salinity contributions to the density become unbalanced, locally weakening the stratification and destabilizing the shear. Meanwhile, high-salinity perturbations found in upflows are advected laterally into downflows, which accelerates them, and similarly for low-salinity perturbations accelerating upflows. Hence while both shear and thermohaline perturbations are stable on their own, they have just the right form to destabilize each other.

Using direct numerical simulations, Radko showed that the thermohaline shear instability gradually decreases $Ri$ below the critical value of approximately 1/4 necessary to trigger a standard stratified shear instability. At this point, Kelvin–Helmholtz billows localized in regions of lowest $Ri$ appear and cause strong mixing, eventually forming a thermohaline staircase whose layer height is directly related to the wavelength of the imposed shear. The layers of this staircase then proceed to merge on a slower time scale, and can grow up to sizes that are commensurate with oceanographic observations regardless of the original wavelength of the shear. This final step is crucial since, without it, the layer formation process would be far too sensitive to the geometrical properties of the assumed shear to be a realistic explanation for diffusive staircases in natural environments.

Of course, much work remains to be done to demonstrate that this new thermohaline shear instability, followed by layer mergers, is indeed the dominant staircase formation mechanism in the high-latitude ocean, and perhaps also in geothermal lakes. Radko acknowledges that his model suffers from some inconsistency, as he assumes the shear to be spatially periodic with a wavelength appropriate for an internal wave field, but also steady, which is not the case for waves, especially since the growth rate of the thermohaline shear instability is several orders of magnitude smaller than the buoyancy frequency. Hence, future work will be necessary to look at the case of an oscillatory shear. Finally, given that the instability growth rate is much longer than a day, one should assess the effects of the Earth's rotation. Nevertheless and despite these caveats, it is clear that Radko's work has opened an entirely new avenue for investigation into the formation of diffusive staircases, and has shown that the fascinating field of double-diffusive layering continues to be full of surprises.

## 3. Long term prospects

The implications of Radko's discovery reach far beyond the scope of diffusive staircases. The notion that one may take two systems that are individually stable but destabilize each other when combined is not entirely new but examples are few and far between, especially if we restrict our attention to hydrodynamic flows. Better known in the astrophysical context are magnetohydrodynamical processes where the addition of a stable magnetic field can destabilize hydrodynamically stable fluids. In addition to the examples listed by Radko, the magnetorotational instability (Balbus & Hawley 1991) perfectly showcases a situation where a constant field (which is always stable if stratification is ignored) can destabilize flows that are Rayleigh stable (i.e. rotating flows stabilized by a gradient of angular momentum). It would be particularly interesting to determine if other examples exist, especially in the non-magnetic case.

Another fundamental implication of Radko's work is that one should always determine the stability of a system holistically rather than by looking at each instability separately. This statement may be self-evident to readers in the fields of geophysical or astrophysical fluid dynamics, who have experience in understanding the intricate ways in which different physical processes interact. While it has been known for a long time that the addition of rotation or moderate shear usually tends to stabilize buoyancy-driven instabilities, Radko has just shown that peculiar situations may exist where the opposite is true. However, the manner in which turbulent mixing is accounted for in state-of-the-art global ocean or atmospheric circulation models (e.g. MITgcm, see Marshall *et al.* 1997) and in stellar evolution models (e.g. MESA, see Paxton *et al.* 2011), is often woefully behind current understanding of hydromagnetic stability. There, instabilities are nearly always treated individually, by successively testing the local stability of a system to a list of independent criteria. Furthermore, it is then customary to attribute a turbulent diffusivity to each instability thus discovered and add them all together. It is obvious that this simplistic two-step parametrization cannot capture the more subtle effects of shear and rotation on mixing processes, but until this work, one may have taken refuge in the thought that as long as the shear is not too strong, or the rotation rate not too large, the error made is acceptable. Radko's paper shows, however, that even a tiny amount of shear can transform a stable system into an unstable one, and that failing to account for the whole dynamical picture can vastly underestimate the amount of mixing present in high-latitude oceans. Whether his new findings will have implications for stellar astrophysics as far reaching as his ground-breaking 2003 paper did is a little too early to know, but I am looking forward to studying the thermohaline shear instability in this context to answer that question.

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