

2

Characteristic properties of detectors

Technical skill is the mastery of complexity while creativity is the mastery of simplicity.

E. Christopher Zeeman

2.1 Resolutions and basic statistics

The criterion by which to judge the quality of a detector is its resolution for the quantity to be measured (energy, time, spatial coordinates, etc.). If a quantity with true value z_0 is given (e.g. the monoenergetic γ radiation of energy E_0), the measured results z_{meas} of a detector form a *distribution function* $D(z)$ with $z = z_{\text{meas}} - z_0$; the *expectation value* for this quantity is

$$\langle z \rangle = \int z \cdot D(z) dz \Big/ \int D(z) dz , \quad (2.1)$$

where the integral in the denominator normalises the distribution function. This normalised function is usually referred to as the *probability density function (PDF)*.

The *variance* of the measured quantity is

$$\sigma_z^2 = \int (z - \langle z \rangle)^2 D(z) dz \Big/ \int D(z) dz . \quad (2.2)$$

The integrals extend over the full range of possible values of the distribution function.

As an example, the expectation value and the variance for a rectangular distribution will be calculated. In a multiwire proportional chamber with wire spacing δz , the coordinates of charged particles passing through the chamber are to be determined. There is no drift-time measurement on

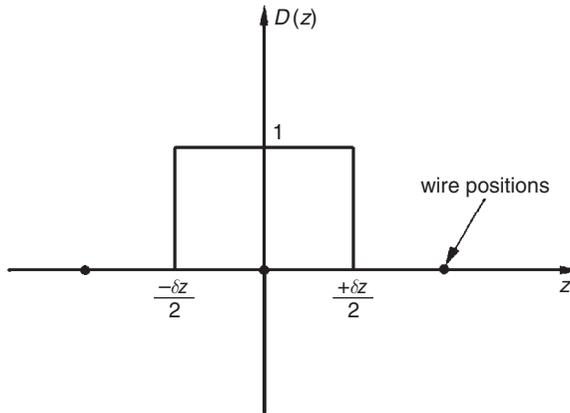


Fig. 2.1. Schematic drawing for the determination of the variance of a rectangular distribution.

the wires. Only a hit on a particular wire with number n_W is recorded (assuming only one hit per event) and its discrete coordinate, $z_{\text{meas}} = z_{\text{in}} + n_W \delta z$ is measured. The distribution function $D(z)$ is constant = 1 from $-\delta z/2$ up to $+\delta z/2$ around the wire which has fired, and outside this interval the distribution function is zero (see Fig. 2.1).

The expectation value for z is evidently zero ($\hat{=}$ position of the fired wire):

$$\langle z \rangle = \int_{-\delta z/2}^{+\delta z/2} z \cdot 1 \, dz \Big/ \int_{-\delta z/2}^{+\delta z/2} dz = \frac{z^2}{2} \Big|_{-\delta z/2}^{+\delta z/2} \Big/ z \Big|_{-\delta z/2}^{+\delta z/2} = 0 ; \quad (2.3)$$

correspondingly, the variance is calculated to be

$$\sigma_z^2 = \int_{-\delta z/2}^{+\delta z/2} (z - 0)^2 \cdot 1 \, dz \Big/ \delta z = \frac{1}{\delta z} \int_{-\delta z/2}^{+\delta z/2} z^2 \, dz \quad (2.4)$$

$$= \frac{1}{\delta z} \frac{z^3}{3} \Big|_{-\delta z/2}^{+\delta z/2} = \frac{1}{3 \delta z} \left(\frac{(\delta z)^3}{8} + \frac{(\delta z)^3}{8} \right) = \frac{(\delta z)^2}{12} , \quad (2.5)$$

which means

$$\sigma_z = \frac{\delta z}{\sqrt{12}} . \quad (2.6)$$

The quantities δz and σ_z have dimensions. The relative values $\delta z/z$ or σ_z/z , respectively, are dimensionless.

In many cases experimental results are normally distributed, corresponding to a distribution function (Fig. 2.2)

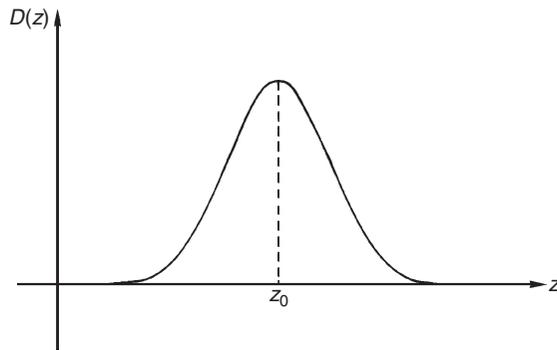


Fig. 2.2. Normal distribution (Gaussian distribution around the average value z_0).

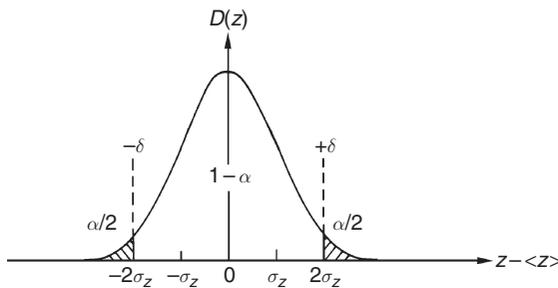


Fig. 2.3. Illustration of confidence levels.

$$D(z) = \frac{1}{\sigma_z \sqrt{2\pi}} e^{-(z-z_0)^2/2\sigma_z^2} . \quad (2.7)$$

The variance determined according to Eq. (2.2) for this *Gaussian distribution* implies that 68.27% of all experimental results lie between $z_0 - \sigma_z$ and $z_0 + \sigma_z$. Within $2\sigma_z$ there are 95.45% and within $3\sigma_z$ there are 99.73% of all experimental results. In this way an interval ($[z_0 - \sigma_z, z_0 + \sigma_z]$) is defined which is called *confidence interval*. It corresponds to a *confidence level* of 68.27%. The value σ_z is usually referred to as a standard error or the standard deviation.

For the general definition we plot the normalised distribution function in its dependence on $z - \langle z \rangle$ (Fig. 2.3). For a normalised probability distribution with an expectation value $\langle z \rangle$ and root mean square deviation σ_z

$$1 - \alpha = \int_{\langle z \rangle - \delta}^{\langle z \rangle + \delta} D(z) dz \quad (2.8)$$

is the probability that the true value z_0 lies in the interval $\pm\delta$ around the measured quantity z or, equivalently: $100 \cdot (1 - \alpha)\%$ of all measured values lie in an interval $\pm\delta$, centred on the average value $\langle z \rangle$.

As stated above, the choice of $\delta = \sigma_z$ for a Gaussian distribution leads to a *confidence interval*, which is called the *standard error*, and whose probability is $1 - \alpha = 0.6827$ (corresponding to 68.27%). On the other hand, if a confidence level is given, the related width of the measurement interval can be calculated. For a confidence level of $1 - \alpha \hat{=} 95\%$, one gets an interval width of $\delta = \pm 1.96 \sigma_z$; $1 - \alpha \hat{=} 99.9\%$ yields a width of $\delta = \pm 3.29 \sigma_z$ [1]. In data analysis physicists deal very often with non-Gaussian distributions which provide a confidence interval that is asymmetric around the measured value. Consequently, this is characterised by asymmetric errors. However, even in this case the quoted interval of $\pm 1\sigma_z$ corresponds to the same confidence level, 68.27%. It should be noted that sometimes the confidence level is limited by only one border, while the other one extends to $+\infty$ or $-\infty$. In this case one talks about a lower or upper limit of the measured value set by the experiment.*

A frequently used quantity for a resolution is the half width of a distribution which can easily be read from the data or from a fit to it. The half width of a distribution is the *full width at half maximum* (FWHM). For normal distributions one gets

$$\Delta z(\text{FWHM}) = 2\sqrt{2 \ln 2} \sigma_z = 2.3548 \sigma_z . \quad (2.9)$$

The Gaussian distribution is a continuous distribution function. If one observes particles in detectors the events frequently follow a *Poisson distribution*. This distribution is asymmetric (negative values do not occur) and discrete.

For a mean value μ the individual results n are distributed according to

$$f(n, \mu) = \frac{\mu^n e^{-\mu}}{n!} , \quad n = 0, 1, 2, \dots \quad (2.10)$$

The expectation value for this distribution is equal to the mean value μ with a variance of $\sigma^2 = \mu$.

Let us assume that after many event-counting experiments the average value is three events. The probability to find, in an individual experiment, e.g. no event, is $f(0, 3) = e^{-3} = 0.05$ or, equivalently, if one finds no event in a single experiment, then the true value is smaller than or equal to 3 with a confidence level of 95%. For large values of n the Poisson distribution approaches the Gaussian.

* E.g., direct measurements on the electron-antineutrino mass from tritium decay yield a limit of less than 2 eV. From the mathematical point of view this corresponds to an interval from $-\infty$ to 2 eV. Then one says that this leads to an *upper limit* on the neutrino mass of 2 eV.

The determination of the efficiency of a detector represents a random experiment with only two possible outcomes: either the detector was efficient with probability p or not with probability $1 - p = q$. The probability that the detector was efficient exactly r times in n experiments is given by the binomial distribution (*Bernoulli distribution*)

$$f(n, r, p) = \binom{n}{r} p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r} . \quad (2.11)$$

The expectation value of this distribution is $\langle r \rangle = n \cdot p$ and the variance is $\sigma^2 = n \cdot p \cdot q$.

Let the efficiency of a detector be $p = 95\%$ for 100 triggers (95 particles were observed, 5 not). In this example the standard deviation (σ of the expectation value $\langle r \rangle$) is given by

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{100 \cdot 0.95 \cdot 0.05} = 2.18 \quad (2.12)$$

resulting in

$$p = (95 \pm 2.18)\% . \quad (2.13)$$

Note that with this error calculation the efficiency cannot exceed 100%, as is correct. Using a Poissonian error ($\pm\sqrt{95}$) would lead to a wrong result.

In addition to the distributions mentioned above some experimental results may not be well described by Gaussian, Poissonian or Bernoulli distributions. This is the case, e.g. for the energy-loss distribution of charged particles in thin layers of matter. It is obvious that a distribution function describing the energy loss must be asymmetric, because the minimum dE/dx can be very small, in principle even zero, but the maximum energy loss can be quite substantial up to the kinematic limit. Such a distribution has a Landau form. The Landau distribution has been described in detail in the context of the energy loss of charged particles (see Chap. 1).

The methods for the statistical treatment of experimental results presented so far include only the most important distributions. For low event rates Poisson-like errors lead to inaccurate limits. If, e.g., one genuine event of a certain type has been found in a given time interval, the experimental value which is obtained from the Poisson distribution, $n \pm \sqrt{n}$, in this case 1 ± 1 , cannot be correct. Because, if one has found a genuine event, the experimental value can never be compatible with zero, also not within the error.

The statistics of small numbers therefore has to be modified, leading to the *Regener statistics* [2]. In Table 2.1 the $\pm 1\sigma$ limits for the quoted event numbers are given. For comparison the normal error which is the square root of the event rate is also shown.

Table 2.1. *Statistics of low numbers. Quoted are the $\pm 1\sigma$ errors on the basis of the Regener statistics [2] and the $\pm 1\sigma$ square root errors of the Poisson statistics*

lower limit		number of events	upper limit	
square root error	statistics of low numbers		statistics of low numbers	square root error
0	0	0	1.84	0
0	0.17	1	3.3	2
0.59	0.71	2	4.64	3.41
1.27	1.37	3	5.92	4.73
6.84	6.89	10	14.26	13.16
42.93	42.95	50	58.11	57.07

The determination of errors or confidence levels is even more complicated if one considers counting statistics with low event numbers in the presence of background processes which are detected along with searched-for events. The corresponding formulae for such processes are given in the literature [3–7].

A general word of caution, however, is in order in the statistical treatment of experimental results. The definition of statistical characteristics in the literature is not always consistent.

In the case of determination of resolutions or experimental errors, one is frequently only interested in relative quantities, that is, $\delta z/\langle z \rangle$ or $\sigma_z/\langle z \rangle$; one has to bear in mind that the average result of a number of experiments $\langle z \rangle$ must not necessarily be equal to the true value z_0 . To obtain the relation between the experimental answer $\langle z \rangle$ and the true value z_0 , the detectors must be calibrated. Not all detectors are linear, like

$$\langle z \rangle = c \cdot z_0 + d, \quad (2.14)$$

where c , d are constants. Non-linearities such as

$$\langle z \rangle = c(z_0)z_0 + d \quad (2.15)$$

may, however, be particularly awkward and require an exact knowledge of the calibration function (sometimes also called ‘response function’). In many cases the *calibration parameters* are also time-dependent.

In the following some characteristic quantities of detectors will be discussed.

Energy resolutions, spatial resolutions and time resolutions are calculated as discussed above. Apart from the time resolution there are in addition a number of further *characteristic times* [8].

2.2 Characteristic times

The *dead time* τ_D is the time which has to pass between the registration of one set of incident particles and being sensitive to another set. The dead time, in which no further particles can be detected, is followed by a phase where particles can again be measured; however, the detector may not respond to the particle with full sensitivity. After a further time, the *recovery time* τ_R , the detector can again supply a signal of normal amplitude.

Let us illustrate this behaviour using the example of a Geiger–Müller counter (see Sect. 5.1.3) (Fig. 2.4). After the passage of the first particle the counter is completely insensitive for further particles for a certain time τ_D . Slowly, the field in the Geiger–Müller counter recovers so that for times $t > \tau_D$ a signal can again be recorded, although not at full amplitude. After a further time τ_R , the counter has recovered so that again the initial conditions are established.

The *sensitive time* τ_S is of importance for pulsed detectors. It is the time interval in which particles can be recorded, independent of whether these are correlated with the triggered event or not. If, for example, in an accelerator experiment the detector is triggered by a beam interaction (i.e. is made sensitive), usually a time window of defined length (τ_S) is opened, in which the event is recorded. If by chance in this time interval τ_S a cosmic-ray muon passes through the detector, it will also be recorded because the detector having been made sensitive once cannot distinguish at the trigger level between particles of interest and particles which just happen to pass through the detector in this time window.

The *readout time* is the time that is required to read the event, possibly into an electronic memory. For other than electronic registering (e.g. film), the readout time can be considerably long. Closely related to the readout time is the *repetition time*, which describes the minimum time which must pass between two subsequent events, so that they can be distinguished. The length of the repetition time is determined by the slowest element in the chain detector, readout and registering.

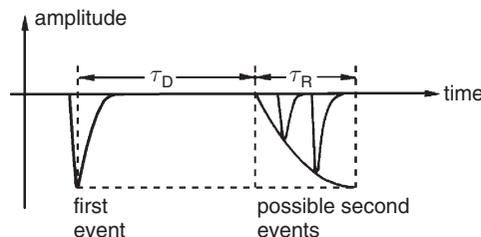


Fig. 2.4. Illustration of dead and recovery times in a Geiger–Müller counter.

The *memory time* of a detector is the maximum allowed time delay between particle passage and trigger signal, which still yields a 50% efficiency.

The previously mentioned time resolution characterises the minimum time difference where two events can still be separated. This time resolution is very similar to the repetition time, the only difference being that the time resolution refers, in general, to an individual component of the whole detection system (e.g. only the front-end detector), while the repetition time includes all components. For example, the time resolution of a detector can be extremely short, but the whole speed can be lost by a slow readout.

The term *time resolution* is frequently used for the precision with which the arrival time of a particle in a detector can be recorded. The time resolution for individual events defined in this way is determined by the fluctuation of the rise time of the detector signal (see Chap. 14).

2.3 Dead-time corrections

Every particle detector has a *dead time* τ_D where no particles after an event can be recorded. The dead time can be as short as 1 ns in Cherenkov counters, but in Geiger–Müller tubes it can account for 1 ms.

If the count rate is N , the counter is dead for the fraction $N\tau_D$ of the time, i.e., it is only sensitive for the fraction $1 - N\tau_D$ of the measurement time. The *true count rate* – in the absence of dead-time effects – would then be

$$N_{\text{true}} = \frac{N}{1 - N\tau_D} . \quad (2.16)$$

Rate measurements have to be corrected, especially if

$$N\tau_D \ll 1 \quad (2.17)$$

is not guaranteed.

2.4 Random coincidences

Coincidence measurements, in particular for high count rates, can be significantly influenced by *chance coincidences*. Let us assume that N_1 and N_2 are the individual pulse rates of two counters in a twofold coincidence arrangement. For the derivation of the chance coincidence rate we assume that the two counters are independent and their count rates are given by Poisson statistics. The probability that counter 2 gives no signal in the

time interval τ after a pulse in counter 1 can be derived from the Poisson distribution, see Eq. (2.10), to be

$$f(0, N_2) = e^{-N_2\tau} . \quad (2.18)$$

Correspondingly, the chance of getting an uncorrelated count in this period is

$$P = 1 - e^{-N_2\tau} . \quad (2.19)$$

Since normally $N_2\tau \ll 1$, one has

$$P \approx N_2\tau . \quad (2.20)$$

Because counter 2 can also have a signal before counter 1 within the resolving time of the coincidence circuit, the total random coincidence rate is [9, 10]

$$R_2 = 2N_1N_2\tau . \quad (2.21)$$

If the signal widths of the two counters are different, one gets

$$R_2 = N_1N_2(\tau_1 + \tau_2) . \quad (2.22)$$

In the general case of q counters with identical pulse widths τ the q -fold random coincidence rate is obtained to be [9, 10]

$$R_q = qN_1N_2 \cdots N_q\tau^{q-1} . \quad (2.23)$$

To get coincidence rates almost free of random coincidences it is essential to aim for a high time resolution.

In practical situations a q -fold random coincidence can also occur, if $q - k$ counters are set by a true event and k counters have uncorrelated signals. The largest contribution mostly comes from $k = 1$:

$$R_{q,q-1} = 2(K_{q-1}^{(1)} \cdot N_1 + K_{q-1}^{(2)} \cdot N_2 + \cdots + K_{q-1}^{(q)} \cdot N_q) \cdot \tau , \quad (2.24)$$

where $K_{q-1}^{(i)}$ represents the rate of genuine $(q - 1)$ -fold coincidences when the counter i does not respond.

In the case of *majority coincidences* the following random coincidence rates can be determined: If the system consists of q counters and each counter has a counting rate of N , the number of random coincidences for p out of q stations is

$$R_p(q) = \binom{q}{p} pN^p\tau^{p-1} . \quad (2.25)$$

For $q = p = 2$ this reduces to

$$R_2(2) = 2N^2\tau, \quad (2.26)$$

as for the twofold chance coincidence rate. If the counter efficiency is high it is advisable to use a coincidence level with p not much smaller than q to reduce the chance coincidence rate.

2.5 Efficiencies

A very important characteristic of each detector is its efficiency, that is, the probability that a particle which passes through the detector is also seen by it. This *efficiency* ε can vary considerably depending on the type of detector and radiation. For example, γ rays are measured in gas counters with probabilities on the order of a per cent, whereas charged particles in scintillation counters or gas detectors are seen with a probability of 100%. Neutrinos can only be recorded with extremely low probabilities ($\approx 10^{-18}$ for MeV neutrinos in a massive detector).

In general, efficiency and resolution of a detector are strongly correlated. Therefore one has to find an optimum for these two quantities also under consideration of possible backgrounds. If, for example, in an experiment with an energy-loss, Cherenkov, or transition-radiation detector a pion–kaon separation is aimed at, this can in principle be achieved with a low *misidentification probability*. However, for a small misidentification probability one has to cut into the distribution to get rid of the unwanted particle species. This inevitably results in a low efficiency: one cannot have both high efficiency and high two-particle resolution at the same time (see Chaps. 9 and 13).

The efficiency of a detector can be measured in a simple experiment (Fig. 2.5). The detector whose unknown efficiency ε has to be determined is placed between two trigger counters with efficiencies ε_1 and ε_2 ; one must make sure that particles which fulfil the trigger requirement, which in this case is a twofold coincidence, also pass through the sensitive volume of the detector under investigation.

The twofold coincidence rate is $R_2 = \varepsilon_1 \cdot \varepsilon_2 \cdot N$, where N is the number of particles passing through the detector array. Together with the threefold coincidence rate $R_3 = \varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon \cdot N$, the efficiency of the detector in question is obtained as

$$\varepsilon = \frac{R_3}{R_2}. \quad (2.27)$$

If one wants to determine the error on the efficiency ε one has to consider that R_2 and R_3 are correlated and that we are dealing in this case

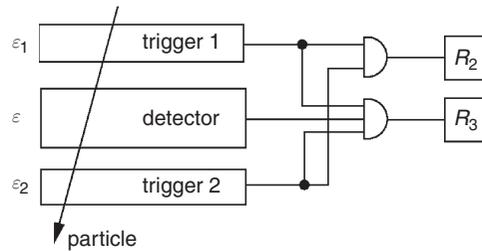


Fig. 2.5. A simple experiment for the determination of the efficiency of a detector.

with Bernoulli statistics. Therefore, the absolute error on the threefold coincidence rate is given by, see Eq. (2.12),

$$\sigma_{R_3} = \sqrt{R_2 \cdot \varepsilon(1 - \varepsilon)} , \quad (2.28)$$

and the relative error of the threefold coincidence rate, normalised to the number of triggers R_2 , is

$$\frac{\sigma_{R_3}}{R_2} = \sqrt{\frac{\varepsilon(1 - \varepsilon)}{R_2}} . \quad (2.29)$$

If the efficiency is small ($R_3 \ll R_2$, $\varepsilon \ll 1$), Eq. (2.28) reduces to

$$\sigma_{R_3} \approx \sqrt{R_3} . \quad (2.30)$$

In case of a high efficiency ($R_3 \approx R_2$, $1 - \varepsilon \ll 1$, i.e. $\varepsilon \approx 1$) the error can be approximated by

$$\sigma_{R_3} \approx \sqrt{R_2 - R_3} . \quad (2.31)$$

In these extreme cases Poisson-like errors can be used as an approximation.

If an experimental setup consists of n detector stations, frequently only a *majority coincidence* is asked for, i.e., one would like to know the efficiency that k or more out of the n installed detectors have seen a signal. If the single detector efficiency is given by ε , the efficiency for the majority coincidence, ε_M , is worked out to be

$$\begin{aligned} \varepsilon_M &= \varepsilon^k (1 - \varepsilon)^{n-k} \binom{n}{k} + \varepsilon^{k+1} (1 - \varepsilon)^{n-(k+1)} \binom{n}{k+1} + \dots \\ &+ \varepsilon^{n-1} (1 - \varepsilon) \binom{n}{n-1} + \varepsilon^n . \end{aligned} \quad (2.32)$$

The first term is motivated as follows: to have exactly k detectors efficient one gets the efficiency ε^k , but in addition the other $(n - k)$ detectors are inefficient leading to $(1 - \varepsilon)^{n-k}$. However, there are $\binom{n}{k}$ possibilities to pick k counters out of n stations. Hence the product of multiplicities is multiplied by this number. The other terms can be understood along similar arguments.

The efficiency of a detector normally also depends on the point where the particle has passed through the detector (homogeneity, uniformity), on the angle of incidence (isotropy), and on the time delay with respect to the trigger.

In many applications of detectors it is necessary to record many particles at the same time. For this reason, the *multiparticle efficiency* is also of importance. The multiparticle efficiency can be defined as the probability that exactly N particles are registered if N particles have simultaneously passed through the detector. For normal spark chambers the multitrack efficiency defined this way decreases rapidly with increasing N , while for scintillation counters it will probably vary very little with N . The multiparticle efficiency for drift chambers can also be affected by the way the readout is done ('single hit' where only one track is recorded or 'multiple hit' where many tracks (up to a preselected maximum) can be analysed).

In modern tracking systems (e.g. time-projection chambers) the multitrack efficiency is very high. This is also necessary if many particles in jets must be resolved and properly reconstructed, so that the invariant mass of the particle that has initiated the jet can be correctly worked out. In time-projection chambers in heavy-ion experiments as many as 1000 tracks must be reconstructed to allow for an adequate event interpretation. Figure 2.6 shows the final state of a head-on collision of two gold nuclei at a centre-of-mass energy of 130 GeV in the time-projection chamber of the STAR experiment [11]. Within these dense particle bundles also decays of short-lived particles must be identified. This is in particular also true for tracking detectors at the Large Hadron Collider (LHC), where a good *multitrack reconstruction efficiency* is essential so that rare and interesting events (like the Higgs production and decay) are not missed. The event shown in Fig. 2.6, however, is a little misleading in the sense that it represents a two-dimensional projection of a three-dimensional event. Overlapping tracks in this projection might be well separated in space thereby allowing track reconstruction.

The multitrack efficiency in such an environment can, however, be influenced by problems of *occupancy*. If the density of particle tracks is getting too high – this will for sure occur in tracking devices close to the interaction point – different tracks may occupy the same readout element. If

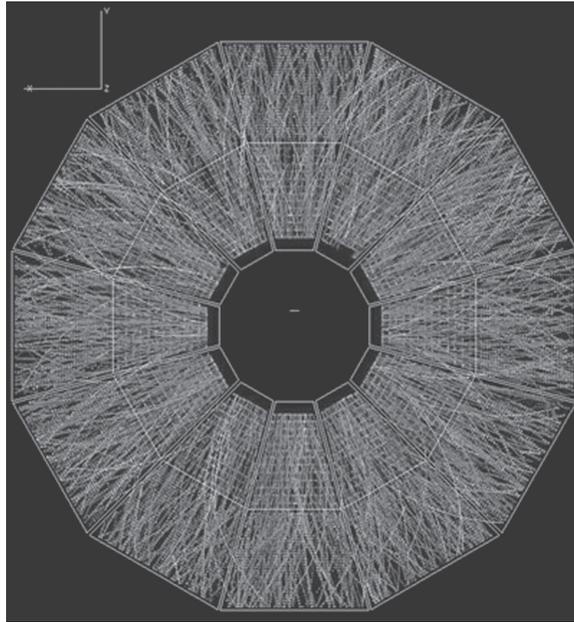


Fig. 2.6. A reconstructed Au + Au collision in the STAR time-projection chamber at a centre-of-mass energy of 130 GeV [11].

the two-track resolution of a detector is denoted by Δx , and two particles or more have mutual distances less than Δx , track coordinates will be lost, which will eventually lead to a problem in track reconstruction efficiency if too many coordinates are affected by this limitation. This can only be alleviated if the *pixel size* for a readout segment is decreased. This implies an increased number of readout channels associated with higher costs. For inner trackers at high-luminosity colliders the question of occupancy is definitely an issue.

Event-reconstruction capabilities might also suffer from the deterioration of detector properties in *harsh radiation environments* (*ageing*). A limited *radiation hardness* can lead to *gain losses* in wire chambers, increase in dark currents in semiconductor counters, or reduction of transparency for scintillation or Cherenkov counters. Other factors limiting the performance are, for example, related to events overlapping in time. Also a possible gain drift due to temperature or pressure variation must be kept under control. This requires an *on-line monitoring* of the relevant detector parameters which includes a measurement of ambient conditions and the possibility of on-line calibration by the injection of standard pulses into the readout system or using known and well-understood processes to monitor the stability of the whole detector system (*slow control*).

2.6 Problems

- 2.1** The thickness of an aluminium plate, x , is to be determined by the absorption of ^{137}Cs γ rays. The count rate N in the presence of the aluminium plate is 400 per 10 seconds, and without absorber it is 576 in 10 seconds. The mass attenuation coefficient for Al is $\mu/\rho = (0.07 \pm 0.01) (\text{g}/\text{cm}^2)^{-1}$. Calculate the thickness of the foil and the total error.
- 2.2** Assume that in an experiment at the LHC one expects to measure 10 neutral Higgs particles of mass $115 \text{ GeV}/c^2$ in hundred days of running. Use the Poisson statistics to determine the probability of detecting
- 5 Higgs particles in 100 days,
2 particles in 10 days,
no Higgs particle in 100 days.
- 2.3** A pointlike radioactive γ -ray source leads to a count rate of $R_1 = 90\,000$ per second in a GM counter at a distance of $d_1 = 10$ cm. At $d_2 = 30$ cm one gets $R_2 = 50\,000$ per second. What is the dead time of the GM counter, if absorption effects in the air can be neglected?

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