

RADIO-ECHO SOUNDING: REFLECTIONS FROM INTERNAL LAYERS IN ICE SHEETS*

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ABSTRACT. The origin and nature of radio-echo internal reflections is the subject of much debate. Calculations of single- and multiple-layer reflection coefficients for dielectric changes in the ice sheet indicate that the observed reflection strength may result from reflections from many thin layers but the zone of thin layers may best be approximated by replacing the zone with a single thicker layer with average dielectric properties. Calculation of reflections from density changes, dust and dirt bands and anisotropy of ϵ indicate density variations as the most likely cause of the internal layering in the uppermost kilometer of ice sheets. Comparison of the depth of internal reflections at "Byrd" station, Antarctica, with the physical properties of the ice measured in a deep core hole at that location support this conclusion.

RÉSUMÉ. *Sondages par écho radio: Réflexions sur les niveaux internes dans les calottes glaciaires.* L'origine et la nature des réflexions internes de l'écho radio sont le sujet de nombreux débats. Les calculs de coefficients de réflexion d'un niveau unique ou de niveaux multiples de changement diélectriques dans la calotte glaciaire montrent que l'intensité des réflexions observées peut résulter de réflexions sur un grand nombre de couches minces mais que la zone de ces couches minces peut être simulée au mieux en remplaçant cette zone par un seul niveau plus épais avec des propriétés diélectriques moyennes. Le calcul des réflexions provenant des changements de densité, de lits de poussière ou de boue et d'anisotropie de ϵ indique que les variations de densité sont la cause la plus probable de la stratification interne dans le kilomètre supérieur des calottes glaciaires. La comparaison de la profondeur des réflexions internes à la Station "Byrd" dans l'Antarctique avec les propriétés physiques de la glace mesurée par un carottage profond dans les mêmes parages confirme ces conclusions.

ZUSAMMENFASSUNG. *Radar-Echolotung: Reflexionen an inneren Schichten in Eisdecken.* Der Ursprung und Charakter inneren Reflexionen beim Radar-Echo wird häufig diskutiert. Berechnungen der Reflexionskoeffizienten für dielektrische Schwankungen in der Eisdecke bei einfacher und mehrfacher Schichtung zeigen, dass die beobachtete Reflexionsstärke durch Reflexionen an vielen dünnen Schichten erzeugt werden kann, dass aber die Zone dünner Schichten am besten durch Ersatz der Zone mit einer einzelnen dickeren Schicht von mittleren dielektrischen Eigenschaften anzunähern ist. Die Berechnung der Reflexion an Dichtesprünge, Staub- und Schmutzbändern und die Anisotropie von ϵ erweisen Dichteschwankungen als die wahrscheinlichste Ursache für die innere Schichtung in dem obersten Kilometer von Eisdecken. Ein Vergleich zwischen der Tiefe innerer Reflexionen an der "Byrd" Station, Antarktika, mit den physikalischen Eigenschaften des Eises, gemessen in einem tiefen Bohrloch an dieser Stelle, stützt diesen Schluss.

INTRODUCTION

Radio-echo reflections from internal layers have been observed over widespread areas of the Antarctic and Greenland ice sheets. The presence of the internal layering has generated a great deal of interest among glaciologists because the layering provides an areal glimpse of the third dimension (depth) of ice sheets previously obtained only at single points through bore holes.

The origin of internal reflections has not been positively determined. There may in fact be several causes of internal reflections and a better understanding of the internal reflections is needed to increase their value as a glaciological tool. Although some of the reflections may originate in a single discrete layer, many apparently arise from the combined reflection coefficient of several layers.

Robin and others (1969) showed that small changes in ice density in an isolated thin layer yield reflection coefficients large enough to produce the observed echoes for layering to depths of approximately 1 000 m. Harrison (1973) suggested that the best explanation for most internal layering is a change in permittivity due to anisotropy in the value of permittivity parallel and perpendicular to the c -axis.

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Paren and Robin (1975) concluded that reflections above a depth of *c.* 1 500 m are due to permittivity changes resulting from density changes and reflections from below *c.* 1 500 m are due to changes in the loss tangent.

Little work has been done to correlate the reflecting layers with physical properties of the ice obtained from cores. In a recent paper Gudmandsen (1975) pointed out that it is difficult to establish a good correlation due to the very small changes in physical properties required to produce reflections and the many variations in some physical properties over distances that are small compared to radar wavelength. Gudmandsen discusses a general comparison between layer echoes and gross changes in the oxygen-isotope ratios.

In this paper calculations of reflection coefficients indicate that multiple layering will not produce a significant increase in the reflection coefficient except that a zone of layers may be considered as a single thick layer with averaged properties. Calculation of reflection coefficients for dust and ash layers and for anisotropy in permittivity show these to be unlikely causes for observed reflections. Finally, a comparison between reflector depth and physical properties of the core at "Byrd" station, Antarctica, shows a good correlation between density variation and reflector depth in the top 1 000 m. This result is in good agreement with Paren and Robin (1975).

REFLECTION COEFFICIENTS—SINGLE THIN LAYER

The complex relative permittivity may be represented by

$$\epsilon^* = \epsilon' - i\epsilon''$$

where ϵ' is the relative permittivity, ϵ'' is the relative loss factor and $i = \sqrt{-1}$. For $\epsilon'' \ll 1$ the index of refraction $n = \sqrt{\epsilon'}$. In this paper $\epsilon^* = \epsilon'$ and changes in the relative permittivity shall be considered. (Changes in ϵ^* due to changes in ϵ'' , or more specifically the loss factor, $\tan \delta = \epsilon''/\epsilon'$ are considered by Paren and Robin (1975).)

The reflection coefficient R for a single layer of ice in terms of the reflection coefficients at the bottom and top interface is

$$R = \frac{R_{23} + R_{12} \exp(2ik_{2z}d)}{1 + R_{23}R_{12} \exp(2ik_{2z}d)} \quad (1)$$

(Brekhovskikh, 1957, p. 50 of the English translation) where R_{23} and R_{12} are the reflection coefficients for the interface between medium 2 and medium 3 (upper space), and medium 1 (lower half-space) and medium 2, respectively, k_{2z} is the vertical wave number in medium 2 and d is the thickness of the layer. For vertical incidence the reflection coefficient at the 1-2 boundary is $(n_1 - n_2)/(n_1 + n_2)$ where n_1 and n_2 are the indices of refraction in medium one and medium two. Similar expressions obtain for any other boundary.

If medium 1 and medium 3 are the same, then $R_{23} = -R_{12}$. For small changes in index of refraction let $n_2 = n_1 + \Delta n$. With $\Delta n \ll n$ the second term in the denominator of Equation (1) can be neglected as being of the order of $(\Delta n)^2$, and $R \approx R_{23}(1 - \exp(2ik_{2z}d))$. If, further, $d \ll \lambda$, $\exp(2ik_{2z}d) \approx 1 + 2ik_{2z}d$, and the expression for R becomes $R \approx -R_{23}(2ik_{2z}d)$. Since $R_{23} = \Delta n/2n$, $R \approx (-i2\pi d/\lambda)(\Delta n/n)$.

This expression is identical to the formula for a single thin layer of Harrison (1973). Robin and others (1969) derive an identical formula in terms of density variation $\Delta\rho$. Using the empirical relationship $n = 1 + 0.85\rho$ between index of refraction n and density ρ (Robin and others, 1969), $\Delta n = 0.85\Delta\rho$ and $2\Delta n/n = 1.7\Delta\rho/n \approx \Delta\rho$. The three equivalent expressions are

$$|R| = \frac{\pi d}{\lambda} \left(\frac{2\Delta n}{n} \right) = \frac{\pi d}{\lambda} \left(\frac{\Delta\epsilon}{\epsilon} \right) \approx \frac{\pi d}{\lambda} (\Delta\rho). \quad (2)$$

Some estimates of single-layer reflection coefficients to be found in the ice sheet are given later.

REFLECTION COEFFICIENTS—MULTIPLE LAYERS

When considering the propagation of a continuous wave with sinusoidal time variation $\exp(-i\Omega t)$, the reflection of waves from multiple layers can be described by a single reflection coefficient. This reflection coefficient can be calculated through repeated application of Equation (1) or alternatively in terms of the input impedance of the layers.

Consider first a single boundary between two media. The tangential components of the field vectors must be continuous across the boundary. Therefore the impedance ζ for layered media is conveniently defined in terms of the tangential components E_t and H_t : $\zeta = E_t/H_t$. This impedance is called the "normal impedance". The normal impedance for a wave whose E vector is parallel to the boundary (perpendicular to the plane of propagation) is defined for each medium as $\zeta = \eta/\cos \theta$ where η is the intrinsic impedance of the medium, $\eta = (\mu_0/\epsilon\epsilon_0)^{1/2}$ and θ is the angle between the direction of propagation and the normal to the boundary. μ_0 and ϵ_0 are the permeability and permittivity of free space. The reflection coefficient for a wave incident in medium 1 at the boundary of medium 2 is

$$R_{12} = \frac{\zeta_2 - \zeta_1}{\zeta_2 + \zeta_1} \quad (3)$$

where ζ_1 and ζ_2 are the normal impedances in medium 1 and medium 2, respectively.

If instead of a half-space with impedance ζ_2 we have a thick sequence of layers with differing impedance and thicknesses we can still use Equation (3) if we let ζ_2 continue to represent the ratio of E_t/H_t at the boundary with medium 1. In other words, the impedance for the thick sequence of layers can be represented by the equivalent impedance of a single layer. This equivalent impedance "looking into" the sequence of layers is defined as the input impedance, $\zeta_1^t = E_t/H_t$. The input impedance at the top of an individual layer ζ_1^t may be found from the input impedance at the bottom of that layer ζ_1^b from

$$\zeta_1^t = \frac{\zeta_1^b - i\zeta_1 \tan \phi_1}{\zeta_1 - \zeta_1^b \tan \phi_1} \quad (4)$$

(Brekhovskikh, 1957), where ζ_1 is the normal impedance of the layer and ϕ_1 is the phase change in the layer, equal to $kd \cos \theta$ where k , θ and d are the wave number, angle of incidence, and thickness for the layer, respectively. Once the input impedance of the sequence of layers is found by the repeated application of Equation (4) from the bottom to the top of the sequence, the reflection coefficient may be found from Equation (3).

Reflection coefficients for a single layer and uniform multiple layers are shown in Figure 1. The reflection coefficient for a single layer increases as the layer thickness increases to thickness $\lambda/4$ where λ is the wavelength. At $\lambda/4$ the reflection from the top and bottom of the layer are in phase with each other and R is twice its half-space value. As the thickness increases further, R decreases until at $\lambda/2$, R is equal to zero. The function repeats periodically with thickness interval $\lambda/2$.

A similar function is obtained for multiple layering for a zone of thin, (thickness $\ll \lambda$) uniform layers with equal layer thickness and spacing. The total thickness (plotted on the abscissa) is the distance from the top of the first layer to the bottom of the last layer of a zone of multiple layers. The zone of layers behaves like a single layer with an impedance equal to the mean value averaged over all the layers in the zone. Hence, in Figure 1 the maximum reflection coefficient for layers of uniform thickness and spacing is half the maximum reflection coefficient for a single layer. This is exactly the value for a half-space.

How well does this averaged layer represent a sequence of layers of non-uniform thickness and spacing? Reflection coefficients for non-uniform layer spacing and thickness were evaluated to determine if a zone of layers with variable spacing and thickness can be approximated by a single layer with averaged properties. If so, then the reflection coefficient for a

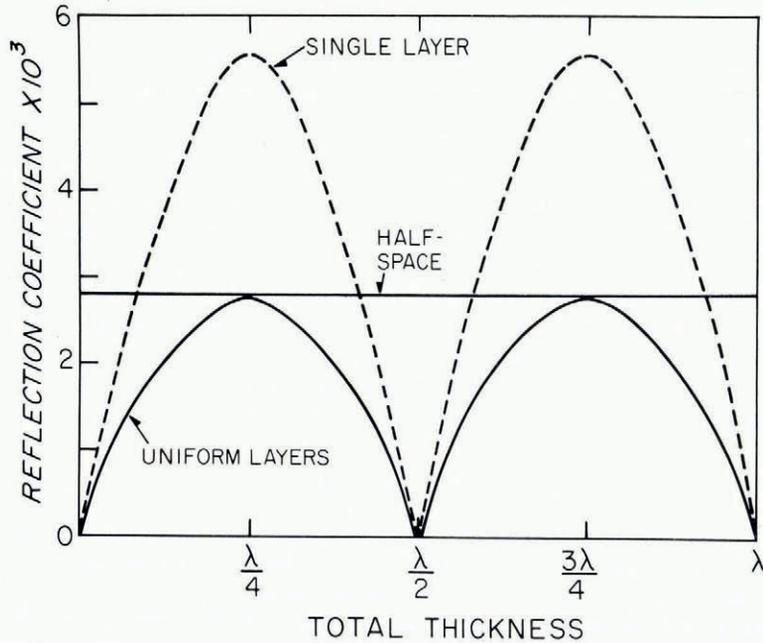


Fig. 1. The magnitude of the reflection coefficient for a single layer and uniform multiple layers plotted as a function of layer thickness with $\Delta n = 0.01$. For uniform multiple layers the thickness and spacing = 0.01λ . The total thickness represents the distance from the top of the first layer to the bottom of the last layer in an increasingly thick zone of layers.

single boundary (half-space) can be used to estimate maximum reflections from multiple layering.

First consider a case where thin layers are randomly spaced in such a manner that all phase delays are equally likely. In this case the mean reflection coefficient for all the layers is expected to be \sqrt{N} times the reflection coefficient for a single layer R_1 , where N is the number of layers. Hence,

$$R = \sqrt{NR_1} \quad (5)$$

We now consider a gated sinusoidal (pulsed) signal. The analysis described above is still applicable as long as the reflected pulse from the upper layers and lower layers interfere with each other. For a time-limited signal with spatial pulse-width P_w , the maximum distance over which the two signals will interfere will be $P_w/2$. Hence, the input impedance for a zone of layers $P_w/2$ in thickness can be used to calculate an upper bound to the reflection coefficient for a pulsed signal. Equation (5) can be applied where N is now the number of layers in a distance $P_w/2$; however, in order for the phases of the various layers to add up in a truly random fashion, the distance between layers must be allowed to vary at least between 0 and $\lambda/2$ ($0 < \phi \leq 2\pi$). As a result, N for a pulsed signal will never in reality become very large when Equation (5) is relevant. For example, for a 35 MHz signal $\lambda \approx 4$ m and the average value for the distance between layers would have to be approximately 1 m. For a pulse width $0.25 \mu\text{s}$ the spatial pulse width P_w is about 40 m. The number of thin layers with a random phase relation in a zone of thickness $P_w/2 = 20$ m is then approximately $N = 20$ and the reflection coefficient R is $\leq \sqrt{20R_1}$.

Now we return to a zone of closely spaced layers. If we assume the layers are thin (0.1–10 cm for example) and the spacing between layers is of the same order of magnitude, then a

large number of layers will occur in the distance $P_w/2$ but the phase relationship between layers no longer varies in a completely random fashion and the expression $R = \sqrt{NR_1}$ no longer holds true. The reflection coefficient for an increasingly thick zone of layers will approach the periodic behavior of uniform layers as shown in Figure 1.

To show that this is so, calculations of reflection coefficients for randomly varying layer spacing and thickness were obtained by high-speed computer. An example is shown in Figure 2 which shows the reflection coefficient for a zone of layers whose thickness and spacing vary randomly between 0.1 cm and 10 cm ($\lambda = 4$ m). The periodic peaks are due to the fact that the zone of layers acts very much like a single layer with ϵ averaged over the zone.

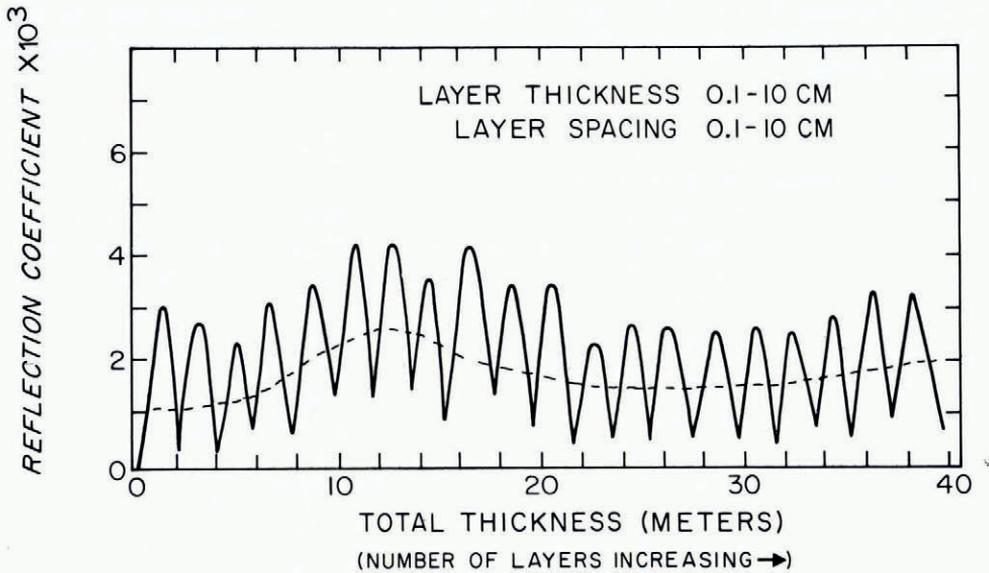


Fig. 2. The reflection coefficient for thin layers with variable thickness and spacing $\ll \lambda$. The variation in the average value (dashed line) is due to the random variation in thickness and spacing between 0.1 and 10 cm. The rapid oscillations arise from the multiples of $\lambda/4$ for the total zonal thickness. $P_w/2 = 20$ m.

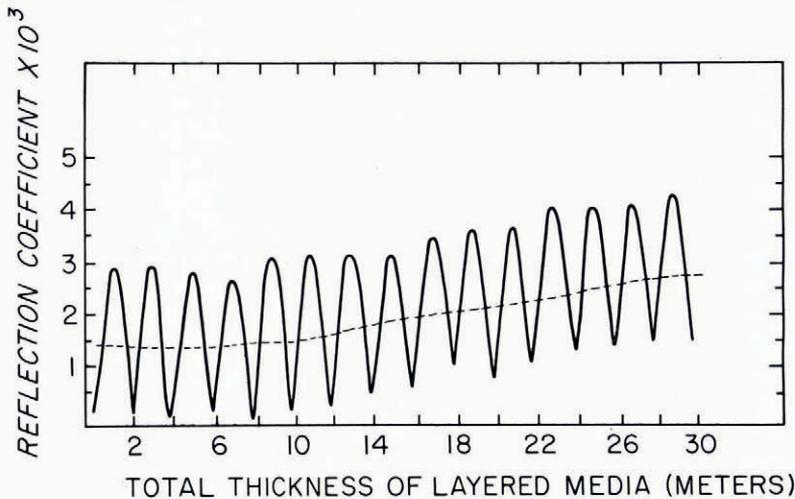


Fig. 3. Reflection coefficients for layers with thickness and spacing uniformly distributed about 3 cm with a standard deviation of 1 cm.

The average reflection coefficient is essentially independent of $P_w/2$. The variations in the average value, or shift of the baseline, is the result of the random variation in thickness and spacing of the layers. The results shown in Figure 2 are typical of more than ten separate models where the thickness and spacing were varied over the same interval (0.1–10 cm). A typical result for a model which may be yet more realistic is shown in Figure 3.

For uniform layers the maximum R (R_{\max}) is equal to the reflection coefficient for a half-space with the same impedance contrast (Fig. 1). For the models calculated with spacing and thickness varying from 0.1 cm to 10 cm, R_{\max} did not deviate from the half-space value of R by more than a factor of two over distances of $P_w/2$ (total thickness ≈ 20 m). The approximation of replacing the zone of thin layers by a single layer with ϵ averaged over the zone is a good one, thus the *value of R for a half-space is a good approximation for the expected value of R_{\max} for multiple layering.*

REFLECTION COEFFICIENT AND PULSE WIDTH

It is significant to note that although the value of R in Figure 2 fluctuates rapidly as the thickness of the layered zone (or P_w) is varied, the value of R_{\max} (or the average value of R) is not very sensitive to changes in P_w . In contrast the value of R from Equation (5) is proportional to P_w . Unfortunately this difference probably cannot be used to determine the nature of the reflecting layers as suggested by Harrison (1973). Harrison observed internal reflection using two different pulse widths (0.25 μ s and 1 μ s) and found the reflected power with the longer pulse width to be 5 dB greater than with the short pulse width. Since the reflection coefficient for specular reflection from a single layer is not a function of pulse width whereas reflection from random multiple layering (Equation (5)) is proportional to pulse width, Harrison concludes, with support from other evidence, that the reflections were produced by multiple reflections according to Equation (5). If, however, the reflecting surface is not perfectly specular, the average reflected power for a single reflecting surface also is a function of pulse width. For example Robin and others (1969) show that the reflected power is proportional to the pulse width and a four-fold increase in pulse width produces a 6 dB increase in reflected power, an effect very similar to that of Equation (5). In this case the signal is integrated over a horizontal rather than a vertical distance.

Although the observed echo strength is a function of P_w this does not necessarily mean that integration is taking place over vertical distances rather than horizontal distances, hence the observed changes in reflected power do not rule out the possibility of reflections of the type shown in Figure 2 which are, on the average, independent of pulse width.

Origin of reflections

The most obvious cause of internal reflections in the relatively pure ice of the Antarctic ice sheet is density variations. Assuming thin layers of bubble-free ice in a generally bubbly ice and calculating bubble volume (bubble pressure) for different depths, Paren and Robin (1975) estimate $\Delta\rho = 0.001$ ($\Delta\epsilon/\epsilon \approx 0.001$) at 800 m. This value is in good agreement with the measured density variations shown in Figure 4 (A. J. Gow, personal communication in 1968). Harrison (1973) estimated density variations using an exponential curve empirically derived by Schytt (1958) from measurements on a core 100 m in length. Although the empirical equation fits the data points reasonably well (there is considerable scatter of the measured densities), extrapolation of his curve to 1 000 m results in estimates of $\Delta\epsilon/\epsilon$ of the order 10^{-12} compared with 10^{-3} found above; hence Harrison's estimates of reflection coefficients due to density variation are several orders of magnitude too low for depths greater than a few hundred meters.

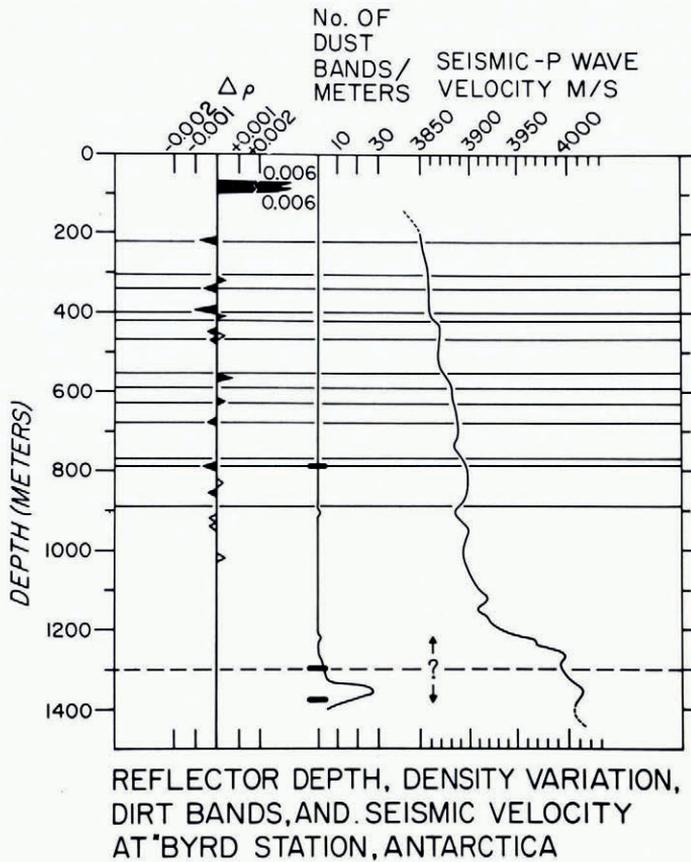


Fig. 4. Comparison of reflector depth (thin horizontal lines) with physical properties of the ice at "Byrd" station, Antarctica. The first column shows measured density variations ($\Delta\rho$) (from A. J. Gow, personal communication in 1968). The open triangles have values of 0.003 or less ($R_p < -80$ dB). Column two shows the distribution of dust bands (solid line) and ash bands (heavy line segments) (from Gow and Williamson, 1971). Variations in seismic velocity (Bentley, 1972) are indicated in column three. The depth of the reflector at c. 1300 m was picked with less precision from a flight profile of Robin and others (1970).

At "Byrd" station, the power reflection coefficient R_p ($R_p = 20 \log_{10} |R|$) for a layer at 1000 m must be about -80 dB in order to be detected (Robin and others, 1969). For a single layer 1 cm thick with $\Delta\epsilon/\epsilon = 0.001$, Equation (2) yields $R_p = -102$ dB. A layer thickness c. 10 cm would be required to produce $R_p = -80$ dB.

In a detailed study of a 411 m ice core from Greenland, Langway (1970) found ice layers and structural features which were about 1–20 mm in thickness. Thick ice layers (3–5 mm) were less numerous than thin ice layers (1–2 mm) but were found more often in *closely spaced groups of two or three individual layers*.

Since layers up to 10 cm thick are probably rare, multiple-layer reflection coefficients or a zone of layers are required to account for the presence of numerous internal reflections. The average value of R_p (dashed line in Figs 2 and 3) for a zone of multiple layering one meter or more in thickness with $\Delta\epsilon/\epsilon$ between layers equal to 0.001 is approximately -72 dB. For a zone 10 cm thick with $\Delta\epsilon/\epsilon = 0.002$ ($\Delta\epsilon/\epsilon = 0.001$ averaged over the zone)

$R_p = -80$ dB as found for a single 10 cm thick layer above. Hence, both zones of 1 m or 10 cm thickness containing density variations of the order of 1 cm thick can produce an ample signal return although a single layer 1 cm thick is not detectable. Reflections from density variations are considered in more detail below when discussing Figure 4.

The ice at "Byrd" station is completely bubble-free below a depth of 100 m (Gow, 1970) and reflections from this depth cannot be explained by density variations. Paren and Robin (1975) have suggested recently that changes in $\tan \delta$ may produce the echo observed below 100 m.

Another mechanism considered for producing internal reflections is the change in dielectric constant within layers of dust or dirt found in the ice sheet. Many layers of volcanic ash and dust have been observed in the ice core from "Byrd" station and described by Gow and Williamson (1971). Only two of these bands occur above a depth of 100 m. An ash band present at 788 m is about 1 mm thick, made up mainly of volcanic glass fragments with concentrations approximately 10^{-4} g/cm³ (A. J. Gow, personal communication in 1974). Assuming a relative dielectric constant for the glass and using the Wiener formula for calculating the dielectric constant of mixture of two dielectrics, a power reflection coefficient R_p of -150 dB is estimated. For a maximum layer thickness of 6 cm and maximum concentrations of 10^{-3} g/cm³ reported by Gow and Williamson (1971) (concentrations were generally very much less than this), R_p for a single layer is still only about -95 dB, not sufficient to produce a signal at the surface.

The large number of dust layers present between 1300–1400 m might produce the reflection at about that depth observed on some flight records near "Byrd" station (Robin and others, 1970, see Fig. 3), but ash and dust bands cannot account for most of the internal reflection observed.

The calculation above was for the ash or dust bands alone. However, these dust and ash bands are accompanied by zones of bubble-free ice up to 10 mm thick and the bubble-free layer alone may produce a more significant reflection coefficient than the dust or ash bands themselves. Even so, R_p for a 10 mm layer of bubble-free ice is only about -100 dB and below 1200 m where most of the ash and dust bands occur the ice is completely bubble-free anyway. Large changes in $\tan \delta$ may be associated with these bands and produce the reflections as suggested by Paren and Robin (1975).

Another possible source of reflections that has been suggested, most recently by Harrison (1973), is anisotropy in the ice. Harrison considers anisotropy to be the most likely source of deep reflections. That anisotropy exists is certain, and changes in ϵ due to anisotropy can produce significant reflections, but we can show that anisotropy is unlikely to produce adequate reflection coefficients for widespread internal layering at "Byrd" station.

Humbel and others (1953) report an anisotropy of $c. 14\%$ in the static dielectric constant. This value has generally been accepted—at least until recently when Wörz and Cole (1969) found different results on measurements of polycrystalline ice with c -axes "nearly parallel". They found "... little if any anisotropy of the static dielectric constant near the melting point, rather than a difference of order 10% reported by Humbel, Jona and Scherrer".

Anisotropy in the optical frequency range (separated from radio frequencies by the infrared absorption bands) is very small; the difference in index of refraction between the extraordinary ray n_e and ordinary ray n_o is only $n_e - n_o = 0.0013$ (Weast, 1975). This corresponds to $c. 0.2\%$ anisotropy in ϵ which compares well with the $c. 0.2\%$ difference in length of the O—O bonds parallel and perpendicular to the c -axis.

The frequency band of interest in radio-echo sounding (radio-frequency region) lies between the principal relaxation absorption band and the infrared absorption band. One unpublished measurement had been made in this region before 1975. At a frequency of 2700 MHz no anisotropy was observed within an experimental accuracy of about 1% (W. Westphal, personal communication in 1974).

Von Hippel and others (1971) list different values of ϵ for single crystals parallel and perpendicular to the c -axis. Although the measurements were made from ice of the same origin, there may have been small differences between samples measured parallel and perpendicular. One of the authors (W. Westphal, personal communication in 1974) states that no inference of anisotropy should be drawn from these data; rather, the 2 700 MHz value ($\Delta\epsilon < 1\%$) should be used.

Johari and Charette (1975) independently concluded that the measurements of Von Hippel and others, which were made for entirely different reasons, do not indicate anisotropy of greater than 1%. They made measurements on polycrystalline ice and single ice crystals (electric field perpendicular to the c -axis) at 35 MHz and 60 MHz and found the values for ϵ' to agree within $\pm 0.2\%$ corresponding to a maximum anisotropy parallel and perpendicular to the c -axis of $c. 1\%$.

If we assume an anisotropy of ϵ equal to 1%, the boundary between a half-space with c -axes all vertical and a half-space with c -axes all horizontal and parallel, resulting in maximum $\Delta\epsilon$, would produce a maximum power reflection coefficient of $c. -52$ dB. The reflection coefficient would be zero if the electric field vector E were perpendicular to the c -axes and -52 dB if E were parallel to the c -axes. Maximum R_p for a single layer 1 cm thick with c -axes all horizontal and parallel in ice otherwise containing all vertical c -axes would be ≈ -79 dB.

Using the value 0.2% for the value between isotropic ice and anisotropic ice with ϵ perpendicular to the c -axis, maximum R_p for an anisotropic layer 1 cm thick with c -axis vertical in normally isotropic ice would be approximately -95 dB. A significant reflection would not be produced.

Small changes in the angle of orientation for ice with c -axes all aligned will produce significant $\Delta\epsilon/\epsilon$ values if we assume that ϵ varies uniformly between ϵ parallel and ϵ perpendicular. For example, if the c -axes are all parallel to the direction of propagation (vertical), E is everywhere normal to the c -axes. Inclining the direction of the c -axes 10° to the direction of propagation would yield $\Delta\epsilon/\epsilon = (\Delta\epsilon_{\max}/\epsilon) \sin 10^\circ \approx 1.5 \times 10^{-3}$ where $(\Delta\epsilon_{\max}/\epsilon)$, the value of anisotropy, is again taken as 1%. This value of $\Delta\epsilon/\epsilon$ is comparable to the value found for density variation. Any reflection produced would be a function of antenna orientation at the surface. If the c -axes were uniformly distributed within a cone forming an angle of 10° with the vertical then

$$\frac{\Delta\epsilon}{\epsilon} = \frac{\Delta\epsilon_{\max}}{\epsilon} \frac{\int_{\theta=0}^{10^\circ} \int_{\phi=0}^{\pi/2} \sin \theta \cos \phi \, d\Omega}{\int_{\theta=0}^{10^\circ} \int_{\phi=0}^{\pi/2} d\Omega}$$

where θ is the angle between the cone and vertical, ϕ is the azimuthal angle, and $d\Omega = \sin \theta \, d\theta \, d\phi$, yielding $\Delta\epsilon/\epsilon \approx 0.75 \times 10^{-3}$ again comparable to the dielectric contrast produced by density changes.

These values are obtained using a value of 1% for anisotropy which might be considered an upper limit. Furthermore, the change in crystal orientation must be sudden (over distance $\ll \lambda$) in order to produce a significant reflection. Such changes in crystal orientation may exist in some glaciers and portions of the ice sheets, but the evidence from the "Byrd" station deep drill core suggests they are not widespread there. The orientation of the c -axes of crystals were observed from thin sections at 40 levels (22 measurements from depths above 1 300 m) throughout the 2 164 m core length taken from the deep drill hole at "Byrd" station (A. J. Gow and T. Williamson, 1976). The c -axes above 300 m depth are quite random. Below 300 m there is a gradual concentration of c -axes toward the vertical to a

depth of approximately 1 200 m where 60% of the c -axes lie within 25% of the vertical. Between 1 200 and 1 300 m there is a rapid increase in the vertical concentration. At 1 300 m about 90% of the c -axes lie within 25° of the vertical.

Ultrasonic compressional wave velocities which have a single-crystal anisotropy of about 7% were measured continuously up the "Byrd" deep drill hole from depths of 1 550 m (Bentley, 1972). The velocity–depth curve observed is shown in Figure 4. The general pattern of increasing c -axis concentration is reflected in the increasing velocities shown. There are several small variations of velocity (± 25 m/s). The velocity changes presumably reflect small changes in crystal orientation. If all the c -axes were uniformly distributed within a cone forming an angle anywhere from 15° to 50° with the vertical, a change in the cone angle of only 5° would produce a 25 m/s change in the ultrasonic velocity (Bennett, 1972, fig. 24). Such a change would produce a $\Delta\epsilon/\epsilon$, again on the order of 4×10^{-4} . The corresponding reflection coefficient for a half-space, $R_p = -80$ dB, would produce a detectable signal.

The seismic velocities shown in Figure 3 were averaged over distances of approximately 2 m and the curve was further smoothed over distances of the order of 20 m, hence changes in crystal orientation larger than indicated by the velocity curve may exist. In order to produce a detectable echo, however, these changes would have to occur over a distance $\ll \lambda$.

Sudden changes in crystal orientation have been observed within the dust and ash bands. The c -axes of ice crystals in these bands are nearly all vertical (A. J. Gow, personal communication in 1974); however, most of the dust and ash bands occur below 1 200 m where all of the ice has strong vertical preferred orientation and so the contrast between the bands and the surrounding ice cannot be very great. The single bands at 788 m and 911 m would produce a significant $\Delta\epsilon/\epsilon$ but the very thin layer (1–2 mm) would produce an R_p of only $c. -100$ dB. For the region near 1 350 m a maximum of 500 layers in a distance of $P_w/2$ (20 m) can be expected. Estimating $\Delta\epsilon/\epsilon = 0.0004$ the maximum reflection coefficient for multiple layering is $R_p \approx -80$ dB. Although anisotropy may produce detectable reflections it seems unlikely that this is the cause for the widespread internal reflections observed in the interior portions of the ice sheet.

Correlation of reflector depth with physical properties of the ice

Radio-echo sounding measurements were made at "Byrd" station, Antarctica, where a 2 164 m core to the bottom of the ice sheet was obtained. A SPRI-Randall Mk II radar (Evans and Smith, 1969) retuned to 50 MHz was used to make a series of measurements aimed at determining the nature of internal reflections. Detailed wide-angle reflection measurements and vertical profiling were conducted near the station. The vertical profiles and the wide-angle reflection measurements were used together to determine the depth of the most prominent reflecting layers within the ice. These reflectors are represented in Figure 4 by the solid horizontal lines. The reflection at a depth of approximately 1 300 m shown by a dashed line was taken from Robin and others (1970, fig. 6). Also shown in the figure are physical properties of the ice measured on the core or in the drill hole. The three mechanisms for producing reflections that were discussed above are shown.

The first column shows density variation as a function of depth. Only 1% of the ice core was sampled for density (approximately 1 cm every 1.0 m). These values were plotted and the density variations were obtained using a graphically produced smooth curve. It is hard to justify correlation of these measurements with the reflecting layers, as so little of the ice is represented. It is shown here, however, as it is the most detailed comparison available to date. The correlation between density variation and reflector depth appears high. The large density variations at approximately 75 and 85 m do not correlate with a reflector because radar reflections could not be picked for depths less than about 200 m. The density

variations shown by open triangles have values of 0.0003 or less ($R_p < -80$ dB) and at depths near 1000 m probably would not be detected. Otherwise, only one density variation does not correlate with a reflecting layer; furthermore, nearly all reflecting layers lie within c. 10 m of a density variation. Because the density was measured at roughly 10 m intervals throughout the core, the measured variations may represent zones of density variation and the correlation of reflector depth with density variation does not imply reflection by single layers.

In column two, the results of Gow and Williamson (1971) are shown. The continuous curve shows the number of dust bands present in the ice column; ash bands are represented by the heavy individual lines. Only one ash band and one dust band occur above a depth of 1200 m. Although reflecting layers are seen at nearly the same depths as these bands, correlation is difficult. In the case of the ash band, a significant density variation occurs at approximately the same depth, and in the case of the dust band, there is an apparent change in crystal orientation.

Column three shows the seismic velocity results of Bentley (1972), indicating changes in crystal orientation. Possible correlations are indicated at 420 m and 890 m; however, both variations of crystal orientation appear quite small.

The correlation between reflector depth and density variation is quite high. Contrarily, the other properties could, at most, account for only a few reflections.

This figure together with the estimates of reflection coefficients obtained for the various mechanisms leave little doubt that density variations are responsible for nearly all the internal echoes in the top 1000 m observed in the vicinity of "Byrd" station.

SUMMARY

Density variations appear to be the most likely cause of the internal layering observed within the top 1000 m of the ice sheets of Greenland and Antarctica. Because the density variations occur over distances small compared to the radar wavelength, the reflections may result from many layers. Zones of multiple layering can be approximated by a single layer with averaged properties. It may never be possible to correlate the internal layering with density variations satisfactorily due to the small spatial variations of density within the ice and the small density contrasts which produce the observed echoes.

If the nature of these internal layers was better known they would serve as a powerful tool to the glaciologist modeling the dynamics of ice sheets. The internal reflection layers most probably represent ancient depositional surfaces. Robin and others (1969) found that the depth of a prominent reflecting layer in Greenland corresponded to a buried surface layer when a steady-state model of dynamics was used for the ice sheet. Internal reflections due to density variations are consistent with the hypothesis of depositional surfaces. Glaciologists should not wait for verification of this hypothesis from the radio engineers and ice physicists but should use (with caution) the assumption of depositional surfaces. In the end, the results the glaciologists obtain when making this assumption may yield the most insight as to the true nature of these reflections.

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