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CUNTZ–KRIEGER ALGEBRAS ASSOCIATED TO SELF-SIMILAR GROUPOIDS

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In this thesis, we study the self-similar action of groupoids on row-finite directed graphs and their associated Cuntz–Krieger algebras. Roughly speaking, if all scales of parts of the object reiterate the whole, then the object is self-similar. For algebraic objects such as groups or groupoids that have this self-similarity property, we simply call them self-similar groups or self-similar groupoids. As an illustration for this self-similarity property, we recall the addition algorithm of integers that we learn from primary school; there is a so-called carrying operation taking place to deal with addition of larger integers. Analogous to this carrying operation, there is a so-called restriction map that encodes the self-similarity property.

In the 1980s, Grigorchuk, Gupta and Sidki introduced the notion of self-similar groups to address the question of whether there exist groups with intermediate growth. Groups act self-similarly on the path-spaces of graphs with a single vertex. For self-similar action on more general directed graphs, Laca, Raeburn, Rammage and Whittaker in 2018 introduced the notion of a self-similar groupoid as a system of partial isomorphisms of the path-spaces of a finite directed graph. They considered self-similar groupoids and their associated C^*-algebras to study the KMS states on the associated dynamical systems. In contrast to their work using the machinery of Hilbert modules and Cuntz–Pimsner algebras, we work solely through generators and relations and the associated Cuntz–Krieger algebras. We also work for a large class of self-similar groupoids focusing on the ideal structure of their associated C^*-algebras.

The main objective of this thesis is to acquire the gauge-invariant $\mathcal{G}$-invariant ideal structure of Cuntz–Krieger algebras pertaining to self-similar groupoids.

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To accomplish this objective, we introduce and scrutinise the Cuntz–Krieger algebras that are linked to self-similar groupoids.

The two main theorems of the thesis are a generalised gauge-invariant uniqueness theorem and a generalised Cuntz–Krieger uniqueness theorem which are new versions in the context of self-similar groupoids. Having these theorems, we are able to completely characterise the gauge-invariant $\mathcal{G}$-invariant ideals of Cuntz–Krieger algebras of self-similar groupoids analogous to results developed (increasingly) for Cuntz–Krieger algebras associated to directed graphs. This work was started in 1980 by Enomoto and Watatani and took several years until it became widely recognised by operator algebraists after rediscoveries by Kumjian, Pask, Raeburn and Renault in 1997. We also determine conditions on the self-similar groupoid under which the associated C^*-algebra is simple.

Some of the research results have been published in [1].

Reference

[1] I. Yusnitha, 'C^*-algebras of self-similar action of groupoids on row-finite directed graphs', *Bull. Aust. Math. Soc.* **108**(1) (2023), 150–161.

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