Preface

Semiclassical methods based on classical solutions play an important role in quantum field theory, high energy physics, and cosmology. Real-time soliton solutions give rise both to new particles, such as magnetic monopoles, and to extended structures, such as domain walls and cosmic strings. These could have been produced as topological defects in the very early universe. Confronting the consequences of such objects with observation and experiment places important constraints on grand unification and other potential theories of high energy physics beyond the standard model. Imaginary-time Euclidean instanton solutions are responsible for important nonperturbative effects. In the context of quantum chromodynamics they resolve one puzzle—the U(1) problem—while raising another—the strong CP problem—whose resolution may entail the existence of a new species of particle, the axion. The Euclidean bounce solutions govern transitions between metastable vacuum states. They determine the rates of bubble nucleation in cosmological first-order transitions and give crucial information about the evolution of these bubbles after nucleation. These bounces become of particular interest if there is a string theory landscape with a myriad of metastable vacua.

This book is intended as a survey and overview of this field. As the title indicates, there is a dual focus. On the one hand, solitons and instantons arise as solutions to classical field equations. The study of their many varieties and their mathematical properties is a fascinating subfield of mathematical physics that is of interest in its own right. Much of the book is devoted to this aspect, explaining how the solutions are discovered, their essential properties, and the topological underpinnings of many of the solutions. However, the physical significance of these classical objects can only be fully understood when they are seen in the context of the corresponding quantum field theories. To that end, there is also a discussion of quantum effects, including those arising from the interplay of fermion fields with topologically nontrivial classical solutions, and of some of the phenomenological consequences of instantons and solitons.

The first half of this book focuses on real-time classical solutions. I focus in particular on three classes of solitons—kinks, vortices, and magnetic monopoles—in one, two, and three spatial dimensions, respectively. Several chapters are devoted to their classical properties and many aspects of their quantum behavior. These are followed by a chapter that discusses the cosmological consequences of domain walls and cosmic strings—the dimensionally extended manifestations of kinks and vortices—and of magnetic monopoles, and the implications of these for proposed high energy theories. Finally, there is a chapter discussing solitons in the BPS limit, including the connections with supersymmetry and duality.

After considering solitons, I turn to Euclidean solutions. Although these are solutions of classical equations, they are associated with tunneling processes that are truly quantum mechanical phenomena. An introductory chapter presenting an overview of this connection is followed by two chapters on Yang–Mills instantons. The first of these is primarily concerned with the mathematical properties of these solutions and their interpretation in terms of vacuum tunneling. Fermions are introduced in the second chapter, which discusses the physical consequences flowing from the instantons. A final chapter describes the bounce solutions and vacuum transitions.

Of necessity, some topics had to be omitted. In particular, Q-balls, nontopological solitons whose existence is based on the possession of a conserved charge rather than on topology, are not covered, nor are skyrmions, a fascinating class of topological solitons.

My goal has been to make the book accessible to advanced graduate students and other newcomers to the field, but also useful for more experienced researchers. I assume that the reader has had an introductory course in quantum field theory and some familiarity with non-Abelian gauge theories, but only the mathematical background of a typical physics graduate student. The homotopy theory needed to understand the topological underpinnings of the solitons is presented and explained. An appendix discusses roots, weights, and other necessary properties of Lie groups and algebras, building on the familiar results associated with SU(2).

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