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Elastic turbulence in space

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Using an analogy between elastic and magnetic effects, Lin *et al.* (*J. Fluid Mech.*, vol. 1000, 2024, R3) use viscoelastic Taylor–Couette flow (TCF) to examine the origin of turbulent mixing in accretion disks. Through direct numerical simulations, the authors find that, unlike the Newtonian case with a similar configuration, turbulence is sustained even at the lowest Reynolds numbers examined and that turbulent mixing is provided through elastic and non-hydrodynamic contributions. By comparing the torque scaling laws obtained with those in magnetized TCF, the authors are able to further support the elastic–magnetic analogy. These findings open new avenues for understanding angular momentum transport and instability mechanisms in both laboratory and astrophysical contexts.

Key words: Taylor–Couette flow, viscoelasticity, turbulent transition

1. Introduction

Taylor–Couette flow (TCF), the flow between two coaxial, independently rotating cylinders, is one of the paradigmatic systems in fluid dynamics. Taylor's seminal work (Taylor 1923) established the correctness of the Navier–Stokes equations and the no-slip condition through linear stability analysis, predicting the instability thresholds of the system and confirming them experimentally. Since then, TCF has become a highly productive laboratory for studying not only instabilities but also topics such as nonlinear dynamics, pattern formation and turbulence (Grossmann, Lohse & Sun 2016). For example, the toroidal cells (known as Taylor rolls) in TCF flow resemble Bénard convection cells, while the waves in spiral turbulence resemble flow structures in binary-fluid convection (Cross & Hohenberg 1993). Insights into one system have often been gained by analysing the other.

The work of Lin *et al.* (2024) builds on this tradition, using TCF with viscous polymers to study solar system formation. The connection between these two systems lies in an unresolved problem in astrophysics: stars and planets cannot form unless they shed most of their original angular momentum, and only turbulent diffusion provides enough mixing for this to happen. However, the origin of turbulence remains elusive. The fact

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that the magnetic Reynolds number seems insufficient to trigger turbulence (Armitage 2011) led to the suggestion that hydrodynamic instabilities could serve as an alternative. Taylor–Couette flow was proposed as a model system to probe this path to turbulence as the independent rotation of the cylinders can be set to resemble a Keplerian-like law, i.e. that the square of a planet's orbital period is proportional to the cube of the semi-major axis of its orbit (Zeldovich 1981; Richard & Zahn 1999). This configuration is termed quasi-Keplerian, as it mimics Kepler's third law by setting the cylinder velocities, but not the entire fluid, to follow this relationship.

Despite this analogy, the behaviour of quasi-Keplerian TCF deviates from expectations. While nonlinear transitions to turbulence are present in pipe or channel flows, they were not observed in quasi-Keplerian TCF, even at high Reynolds numbers. The initial experiments by Ji *et al.* (2006) with carefully controlled boundary conditions showed an absence of significant turbulence, which led to renewed interest in the problem, with some experiments showing increased mixing and others showing none (Ji & Goodman 2023). It is now well established that, in the quasi-Keplerian regime, TCF does not exhibit a nonlinear transition to turbulence at high Reynolds numbers. Any observed turbulence is attributed to the presence of end plates required to physically close the experiment (Lopez & Avila 2017; Ji & Goodman 2023). This surprising result prompted a return to an earlier idea: a magneto-hydrodynamic origin for the turbulence.

In the presence of a magnetic field, cylinders rotating in a quasi-Keplerian manner become linearly unstable through the magneto-rotational instability (MRI) (Chandrasekhar 2013). However, this instability was only expected to occur at very large magnetic Reynolds numbers, which are difficult to achieve experimentally. The work of Lin *et al.* (2024) seeks to bridge this gap by studying the MRI through its analogy with the elasto-rotational instability in non-Newtonian TCF, based on the ideas of Ogilvie & Proctor (2003) and Ogilvie & Potter (2008). As they explain, the viscoelastic stresses in dilute polymeric fluids, modelled by dumbbells, and the Maxwell stresses in electrically conducting fluids, governed by magnetohydrodynamics, have mathematically similar terms (Vieu & Mutabazi 2019). This similarity allows the use of direct numerical simulations (DNS) of polymeric fluids to study quasi-Keplerian TCF and gain insights into stellar formation. Additionally, the Reynolds numbers required for this analysis can be achieved (and likely exceeded) in laboratory settings.

2. Results

Lin *et al.* (2024) conduct a series of DNS of a Taylor–Couette system using a polymeric fluid modelled with the finite extensible nonlinear elastic-Peterlin (FENE-P) model. They vary the Reynolds number (Re) between 10^2 and 10^4, and fix the Weissenberg number at $Wi = 30$. While this configuration results in laminar flow for a Newtonian fluid, all simulations with the polymeric fluid show turbulence, even at the lowest Reynolds number examined. This has some immediate consequences: since the turbulence is sustained by the energy input from the torque on the cylinders, the system's torque must exceed that of the laminar state. This implies that angular momentum is transported from the inner to the outer cylinder through mechanisms other than viscosity. Remarkably, the authors demonstrate that this transport is primarily due to elastic stresses, rather than hydrodynamic or convective transport (cf. figure 1*a*). This finding emphasizes that even in turbulent quasi-Keplerian TCF, hydrodynamic turbulence does not contribute to angular momentum mixing.

The authors also analyse the structures present in the flow, finding that they differ significantly from the large-scale rolls commonly observed in TCF. These structures

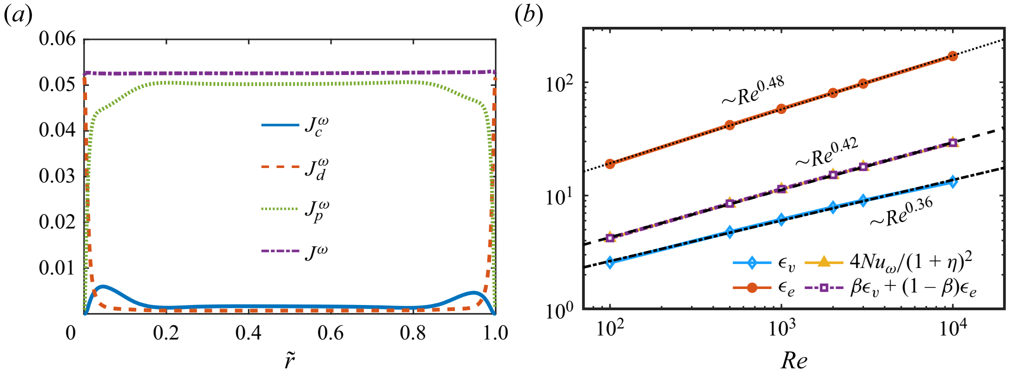

Figure 1. (*a*) Different contributions to angular velocity transport J^{ω} (analogous to the torque) for $Re = 10^4$, $Wi = 30$ across the cylinder gap: J_d^{ω}, viscous; J_p^{ω}, elastic; and J_c^{ω}, convective. The viscous contribution dominates close to the cylinders, while the elastic contribution dominates the bulk. (*b*) Viscous (ϵ_v) and elastic (ϵ_e) contributions to the total dissipation (shown in purple) as a function of Reynolds number. The elastic component can be seen to be much larger than the viscous one, and to follow a $Re^{0.48}$ scaling law.

decrease in axial size as the Reynolds number increases. They examine the boundary layer scaling, showing that the extent of the boundary layers diminishes approximately as $\sim Re^{-1/2}$, which is consistent with the behaviour of a Prandtl–Blasius boundary layer. The authors then elucidate scaling laws for the torque, non-dimensionalized as a Nusselt number ($Nu_{\omega} = T/T_{pa}$, where T is the torque and T_{pa} is the torque for the purely azimuthal state). They compare these scaling laws with those found in Newtonian TCF and magnetized TCF, noting that the results closely match those obtained by Mishra, Mamatsashvili & Stefani (2023) for TCF destabilized by the standard MRI, with a scaling of $Re^{0.4-0.5}$. In addition, they note that these scaling laws are less steep than those observed in Newtonian TCF (Grossmann *et al.* 2016). Finally, the authors break down the contributions to the torque, distinguishing between those arising from viscous dissipation and those from elastic dissipation. They show that the scaling laws hold quite well across the examined parameter space, and that elastic dissipation scales more steeply with Reynolds number than viscous contributions (cf. figure 1*b*). However, they note that the full range of applicability of these scaling laws and their origins are still unclear.

3. Outlook

Research on viscoelastic TCF dates back to the early 1990s, but DNS and experimental studies in the turbulent regime have primarily emerged in the last decade (Song *et al.* 2023). The work by Lin *et al.* (2024) serves as an important complement to an earlier study by the same research group (Song *et al.* 2021), which investigated viscoelastic turbulent TCF with pure inner cylinder rotation. In this configuration, transport occurs through both elastic and hydrodynamic mechanisms with the latter predominating at high *Re*. Future experiments and simulations across a broader range of Reynolds numbers and cylinder rotation configurations will provide valuable insights into how the scaling laws for torque evolve, and in particular, what exponents will be obtained for the viscous, hydrodynamic and elastic contributions across the large parameter space of TCF. Additionally, it will be important to explore the dominant flow structures for different driving parameters, the behaviour of boundary layers as they become turbulent and other intriguing questions, such as the nature of pure ‘viscoelastic’ TCF.

Meanwhile, the astrophysical community continues to explore the nonlinear dynamics beyond the onset of the MRI in accretion-disk-like systems. This research is ongoing, both through magnetic TCF realizations (Wang *et al.* 2022) and simulations (Chan, Piran & Krolik 2024). Other mechanisms, such as stratification, are also being studied to address the challenges posed by low magnetic Reynolds numbers (Sandoval *et al.* 2024). Further exploration of viscoelastic TCF may lead to more discoveries by analogy, as suggested by the authors' comparison with Mishra *et al.* (2023). A persistent challenge, however, remains the need for fluid dynamicists and astrophysicists to speak a common language. While TCF has at times served as a bridge between these fields, continued effort from both communities is needed to maintain and expand this tradition.

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