(5-1) There is a sharp for an inner model with ω^2 many Woodin cardinals.
(5-2) All $<\omega^2$-$\mathbf{\Pi}^1_1$ games on natural numbers of length ω^3 are determined.
(5-3) There is a sharp for $L(\mathbb{R}, \mu)$ and $L(\mathbb{R}, \mu) \models$"$\mathsf{AD} + \omega_1$ is $\mathbb{R}$-supercompact," where μ is the club filter on $\wp_{\omega_1}(\mathbb{R})$.

Trang also obtained such equivalence for a sharp for an inner model with ω^α many Woodin cardinals for any $\alpha < \omega_1$ by introducing generalized Solovay models. Supercompactness of ω_1 seems important beyond determinacy of fixed countable length games too. Steel proved several results on the relation between the theory (2-2) and games ending at the first Σ_n-admissible relative to the play (cf. J. R. Steel, *Long games*, ***Games, Scales, and Suslin Cardinals: The Cabal Seminar Volume I*** (A. S. Kechris, B. Löwe, J. R. Steel, editors), Cambridge University Press, Cambridge, Lecture Notes in Logic, 31, 2008, pp. 223–259). Also, based on Neeman's consistency proof of long game determinacy (cf. I. Neeman, ***The Determinacy of Long Games*** De Gruyter, Berlin, De Gruyter Series in Logic and its Applications, 7, 2004, xii+317 pp.), Woodin showed that, assuming a sharp for an inner model with a Woodin limit of Woodin cardinals, it is consistent that ZFC + all games on natural numbers of length ω_1 with payoff sets that are definable from real and ordinal parameters are determined. He then used such determinacy to prove the aforementioned theorem on CM^+.

Many questions about the supercompactness or strong compactness of ω_1 are still open and seem crucial for a proper understanding of the connection between inner models and determinacy axioms. The four papers under review could be good starting points for anyone interested in tackling such questions.

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Christopher Pincock. ***Mathematics and Explanation***. Elements in the Philosophy of Mathematics. Cambridge University Press, Cambridge, UK, 2023, 80 pp.

Can mathematics play an explanatory role in science? Can mathematics play an explanatory role 'internally', that is, within mathematics itself? Although these questions may be seen as referring to two independent areas of research, namely that concerning mathematical explanations in science (also called by Pincock 'genuine mathematical explanations') and that relative to mathematical explanations in pure mathematics, they both revolve around the idea that mathematics can disclose the reasons why something (an empirical phenomenon or a mathematical fact) is the way it is. Thus, philosophers of mathematics usually consider them as pertaining to a broader, unified field of research focused on the nature of mathematical explanation. In ***Mathematics and Explanation***, Christopher Pincock provides a comprehensive and critical overview of this field of research. Furthermore, he goes beyond existing studies on mathematical explanation by proposing novel ideas and questions.

The book is organised into five sections. The first section serves as a brief introduction in which Pincock outlines the general architecture of ***Mathematics and Explanation***. The introductory section also provides insight into a crucial aspect of Pincock's approach to the philosophy of mathematics, namely his attempt to 'attend carefully to mathematical and scientific practice in philosophical work' (p. 2). What does this attempt amount to? In line with his previous works (e.g., his 2012 book ***Mathematics and Scientific Representation***), Pincock pursues an epistemology of mathematics that is sensitive to actual mathematical and scientific practice. His work on mathematical explanation can therefore be situated within a broader trend in current philosophy of mathematics, known as the 'philosophy of mathematical practice', which opts for a bottom-up methodology that draws particular attention to the way(s) in which mathematics is actually practiced (see P. Mancosu (Ed.), ***The Philosophy of Mathematical Practice***. Oxford University Press, 2008).

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Monism about explanation is the view that all explanations share a feature that makes them explanatory. Sections 2 and 3 are devoted to an assessment of 10 influential monist accounts of explanation in both mathematical and scientific contexts. These proposals, ranging from well-known accounts like Hempel's DN model to more recent ones such as Povich's ontic counterfactual account, are evaluated to determine their ability to make sense of the explanatory role of mathematics. Pincock's strategy is to take for granted five principles that any legitimate theory of mathematical explanation should conform to and use them to assess the 10 accounts. The result of his test is that none of these models meets all five principles, indicating the need for a different approach.

In Section 4, Pincock moves to an examination of the opposite view: pluralism about explanation (also termed 'explanatory pluralism' by Pincock). A pluralist about explanation holds that explanations come in several types and that 'the best way to make sense of the variety of explanations in science and mathematics is to posit two or more explanatory relevance relations' (p. 34). Such explanatory relevance relations are taken by Pincock as relations that 'connect facts in the world when one fact explains the other' (*Ibid.*). In this section, Steiner's and Lange's pluralist accounts of mathematical explanation in pure mathematics and in science are examined. Pincock's analysis, grounded in case studies of explanatory proofs and genuine mathematical explanations (i.e., mathematical explanations that are valued as such by mathematicians and scientists), reveals limitations in both proposals: neither view successfully identifies the explanatory nature of these cases. Consequently, Pincock advocates for a more radical form of pluralism—'brute' pluralism—which posits additional explanatory relevance relations that cannot be subject to a reductive analysis. This perspective, as Pincock observes, is also motivated by 'the assumption that explanations have distinctive cognitive and epistemic roles in both mathematics and science'(p. 57).

In Section 5 Pincock focuses on the relationship between mathematics and physics. Building on the notion of explanatory autonomy discussed by Putnam (see H. Putnam, *Philosophy and Our Mental Life*. In ***Mind, Language and Reality: Philosophical Papers***, Cambridge University Press, pp. 291–303), he defends the idea that mathematics is weakly, but not strongly, autonomous from physics. To say that a science is weakly autonomous from physics is to say that it provides explanations of some phenomenon P that are *better in some respect* than the explanations of P provided by physics (strong autonomy, on the other hand, requires that a science provides explanations of P and physics *cannot* explain the same phenomenon). According to Pincock, weak autonomy holds for mathematics because there exist mathematical explanations of some phenomena that are better than their physical competitors in two respects: generality (mathematical explanations apply to a class of physical systems and not only to a particular system) and robustness (mathematical explanations are not affected if we introduce small structural variations in the physical system).

Pincock's discussion of the weak autonomy of mathematics is per se a major contribution to the current debate on mathematical explanation, since it spotlights two explanatory virtues that make some mathematical explanations better than their non-mathematical alternatives. Nevertheless, the significance of such discussion is even more pronounced in the final part of the book, where Pincock turns to the ontological debate between platonists and nominalists. Such debate is presently centered on the enhanced indispensability argument (EIA), whose basic idea is the following: since mathematical entities participate in our best scientific explanations on an epistemic par with unobservable entities, and since inference to the best explanation (IBE) supports rational belief in unobservable entities, we ought to be committed to the existence of those mathematical objects that play an indispensable explanatory role in empirical science. In response to this argument, Pincock maintains that the weak autonomy of mathematics is not enough to motivate a platonist interpretation of mathematics via EIA. And this is because the explanatory virtues (generality and robustness) that make genuine mathematical explanations better than their non-mathematical alternatives do not depend on the intrinsic character of mathematical objects. Thus, in contrast to what is claimed by proponents of EIA, the explanatory value of genuine mathematical explanations must be seen as disconnected from a platonist interpretation of mathematics in terms of abstract mathematical objects.

In the last pages, Pincock extends his discussion of EIA by focusing on the appeal to IBE in support of platonism. He first presents a form of the pessimistic induction argument against the adoption of traditional explanatory virtues like simplicity, generality and depth, as triggers for IBE: since the history of science has shown that the adoption of these particular virtues in IBE has led to failure, we have no reason to believe that these virtues can be taken seriously when used to guide our inferences via IBE. Next, he introduces two new explanatory virtues (conservativeness and modesty) and shows that they provide a defensible, although restricted, form of IBE. This restricted form of IBE is very useful in the hands of the scientific realist, since it bypasses the difficulties arising from the adoption of traditional explanatory virtues. However, as Pincock shows, it cannot be employed by the platonist because genuine mathematical explanations do not meet the two special explanatory virtues that characterise it.

Written by a key actor in the debate on mathematical explanation, ***Mathematics and Explanation*** is a remarkable book. In less than 100 pages, it brings together, synthesises and critically assesses the most influential studies in this field. Furthermore, it adds to the current literature on mathematical explanation by proposing a solid defence of explanatory pluralism and a sensitive diagnosis of why the platonist's use of EIA remains unpersuasive. Surely, the extent to which the pluralist perspective can encompass the diverse array of mathematical explanations presented by scientists and mathematicians without being too permissive remains an open question. It also remains an open question whether the platonist can overcome the difficulties highlighted by Pincock and find a more effective way to support her stance through EIA. Nonetheless, there is no doubt that Pincock's arguments against monism and in favour of a broad-ranging explanatory pluralism, as well as his deep analysis of the viability of using genuine mathematical explanations and IBE to support platonism, have far-reaching implications and open new and exciting avenues for future research.

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