J. Austral. Math. Soc. (Series A) 51 (1991), 343–380

OBITUARY
KURT MAHLER, 1903–1988

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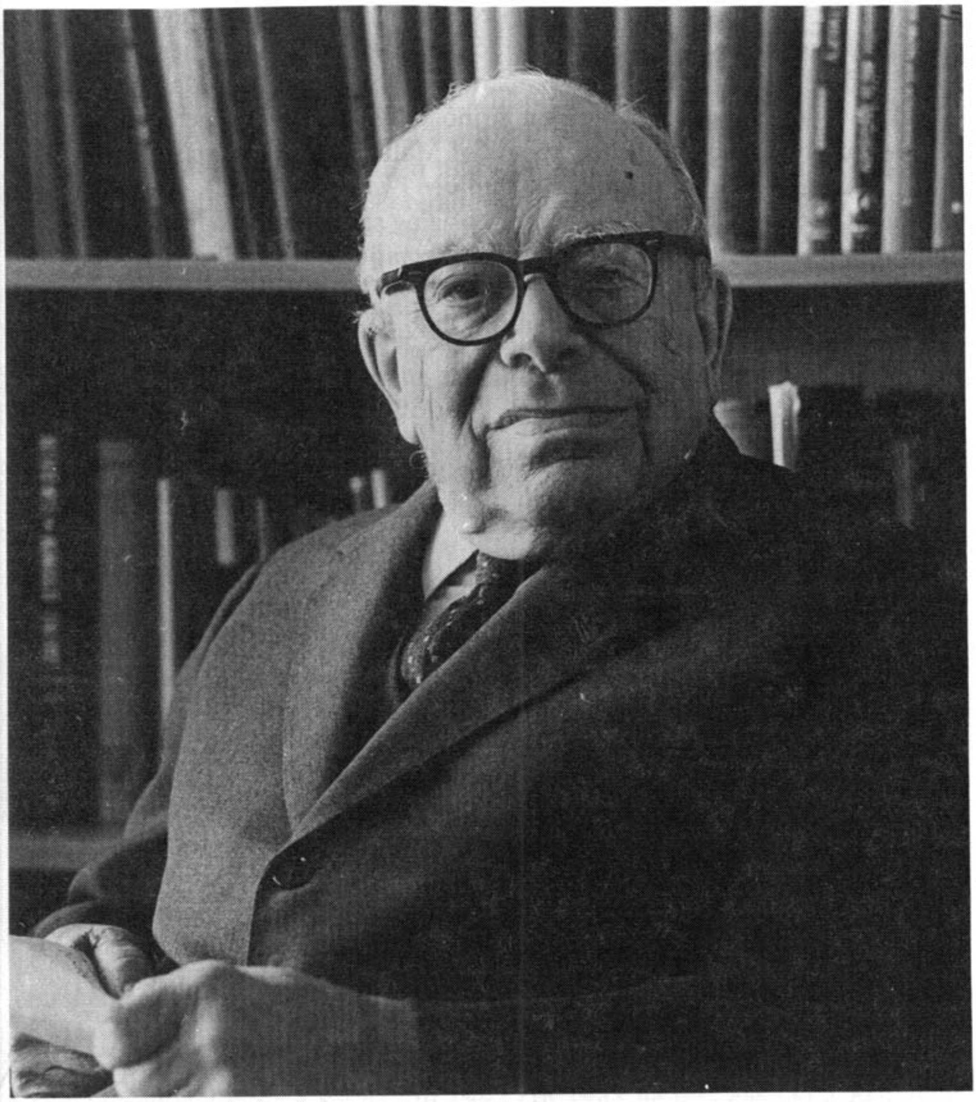

https://doi.org/10.1017/S1446788700034558 Published online by Cambridge University Press

1. Introduction

Kurt Mahler FRS, FAA died on 26 February, 1988 after a long and distinguished career devoted primarily to the theory of numbers. Mahler's personal papers contain extensive biographical remarks most of which have not been published previously. It seemed attractive to allow him to speak for himself. He does that here in the accompanying 'Fifty years as a Mathematician II' taken from a typescript written principally in 1971. There are also a number of previously published papers of Mahler with biographical content: 'Fifty Years as a Mathematician' (*J. Number Theory* **14** (1982), 121–155) is a version of the present attachment emphasising some mathematical contributions. There is also 'How I became a mathematician' (*Amer. Math. Monthly* **81** (1974), 981–83) and an essay (written in 1923) 'Warum ich eine besondere Vorliebe für die Mathematik habe' (*Jber. d. Deutschen Math.-Verein.* **85** (1983), 50–53).

Kurt Mahler was born on July 26, 1903 at Krefeld am Rhein, a town of some 100,000 inhabitants on the western side of the Rhine close to the Dutch border. Kurt and his twin sister Hilde were the youngest children of Hermann Mahler (1858–1941), owner of a small printing firm, and Henriette née Stern (1860–1942).

The family had no academic tradition or connections though like their father, the four surviving children were avid and indiscriminate readers. It seems doubtful whether Kurt would have reached University if he had been in good health.

At age 5 he contracted tuberculosis which severely affected his right knee. As a result his school attendance for the following years was only intermittent. At age 7 his knee was operated on but for many years he had an open wound and the leg became stiff bent at the knee, very much hindering his walking.

Because of his ill-health Kurt had only some 4 years of formal schooling by his 14th birthday; however he had had some private tuition at home. With the intention of becoming a *Feinmechaniker* (precision engineer) he attended elementary technical schools and there he became fascinated with mathematics and bought himself books from which to teach himself.

He chose to accept an apprenticeship at a machine factory in Krefeld in the hope of being allowed to enter a *Technische Hochschule* to study mathematics—thereby avoiding the difficult *Abiturienten-Examen* necessary for matriculation to a University.

Mahler worked for almost 3 years as an apprentice at the machine factory, spending the first year in the drawing office. (Later, the skills he acquired would be useful: see the drawings in papers of L. J. Mordell in the period 1941–45.) By about 1920 he was teaching himself about analytic functions, elliptic functions, modular functions, number theory and non-Euclidean geometry—guided in his choice of books by little more than the advertisements in Teubner's mathematics books.

He was in the habit of writing little articles about the mathematics he had read. Without his knowledge Mahler's father gave some to Dr. Josef Junker the director of the local *Realschule* and a former student of Cristoffel. Dr. Junker brought the work to the attention of Felix Klein who passed them on to C. L. Siegel who was then doing research at Göttingen; and on their recommendation Mahler was prepared for the *Arbiturienten-Examen.*

He was admitted to the University of Frankfurt am Main in 1923. He stayed there until early 1925 and from then, assisted by a research fellowship from 1930, he studied and did research at Göttingen until 1933. His thesis 'Ueber die Nullstellen der unvollständigen Gammafunktionen' [2] was submitted at Frankfurt. During the years at Göttingen Mahler developed a new method (now known as 'Mahler's Method') in transcendence theory [4],[7],[8]; found his celebrated classification of transcendental numbers [11], [9]; and introduced and developed diophantine approximation and transcendence theory in p-adic fields [14].

With the rise of Hitler in 1933, Mahler realised he would have to leave Germany. He visited Koksma and Popken in Amsterdam and briefly held a fellowship with Mordell in Manchester. Eventually van der Corput arranged a two year fellowship at Groningen where he lectured on diophantine approximation and Minkowski's geometry of numbers. Towards the end of that stay he had a collision with a bicycle rider which reactivated the tuberculosis in his knee. He confirmed that he could still do mathematics after his long convalescence by proving [46] the transcendence of $0.1234567891011121314\ldots$.

Mahler returned to Manchester in the autumn of 1937 and remained there until 1963. In 1940, while interned as an enemy alien, he was awarded the DSc from the University. In 1941 he received a three year appointment as an assistant lecturer and then was successively promoted to lecturer (1944–47), senior lecturer (1947–49), reader (1949–52) and finally to a chair in mathematical analysis in 1952. The early forties saw the founding, in part by Davenport and Mordell, of the modern theory of the geometry of numbers to which, then and later, Mahler made fundamental contributions.

He became a British subject in 1947. In 1948 he was elected a Fellow of the Royal Society. Mahler received the Senior Berwick Prize of the London Mathematical Society in 1950 and was made an honorary member of *het Wiskundig Genootschap* (the Dutch mathematical Society) in 1957.

From 1949 Mahler travelled widely, with visits to Jerusalem, Brussels, Vienna and many journeys to the United States—including extended stays at the Institute for Advanced Study, Princeton; at the University of Colorado; and at the University of Notre Dame. In 1962, he visited the Australian National University in Canberra from September to December.

Mahler was appointed Professor of Mathematics at the Institute of Advanced Studies of the ANU and took up the post in September 1963 holding it until he turned 65 in 1968. He was elected a Fellow of the Australian Academy of Science in 1965.

After his retirement, Mahler accepted a professorial appointment at Ohio State University, Columbus holding it until 1972 when he returned as an honorary fellow to the IAS, Canberra. He was awarded the de Morgan Medal of the London Mathematical Society in 1971. He took Australian citizenship in 1974. In 1977 he was awarded the Lyle Medal of the Australian Academy of Science and, in Frankfurt, he was honoured by the celebration of the golden jubilee of his doctorate. Mahler was elected an honorary member of the Australian Mathematical Society in 1984.

From 1984 severe angina prevented his travelling and he did not again leave Canberra. Mahler continued his mathematical work well into his eighties and wrote some 35 papers after returning from Columbus; he submitted a final manuscript a few days before his death. He had published over 200 papers (one in Chinese [96]) and four books. His influence on Australian mathematics is evident in Australia's continuing strong reputation in transcendence theory and related fields.

Mahler had accepted an appointment in Szechuan, China but could not take it up because of the outbreak of war. He had begun to learn Chinese and that study remained an important interest and hobby. He was an excellent photographer; many of his pictures adorn University House at the ANU where he lived for more than twenty years. His non-mathematical reading comprised mostly science fiction and history.

An abiding memory of Kurt Mahler is him sitting in the centre of the front row at lectures and, as the time for the break came, of him carefully collecting his string-bag, his hat and stick and beginning to make his way out, apparently regardless of the state of the lecture. At Kurt's funeral Bernard Neumann rather infelicitously began his oration at exactly the appointed moment with the words: 'Kurt Mahler was always dead on time ... '.

Kurt Mahler may have seemed to be, but was not a forbidding man. When I came to see him I would be treated to his political views; I might then be shown some photos to admire the technical expertise required to produce them and have explained to me some principles of Chinese. Then would come a summary of some recently read historical book. Sometimes I would be told of his current mathematical work and would be criticised for preferring HP calculators to his TI[1]. Finally, the conversation would turn to fantasy and science fiction and I could join in. He enjoyed my sending him books from my collection though he found that old favourites from the fifties were rarely as good as he had remembered. Amongst others, John Brillhart (Arizona) kept Kurt supplied with recent science fiction books as they appeared in the States. Very occasionally we would talk about my mathematics. His response typically seemed to be "You'll find it in Hasse[2] (or Bachmann or …)". When once I complained at length how my honest efforts had as yet produced no success he issued the useful dictum: "If you want to get chopped meat out of the meatgrinder you must first put meat into the meatgrinder".

Mahler's 'final retirement' in Canberra in 1972 was anything but that. He continued his work publishing at a rate and of a quality that should be the envy of most 'working' mathematicians. He particularly enjoyed his programmable pocket computer and some of his most recent work on digital properties of numbers relied on extensive hand calculations; see, for example, [217] and Kurt Mahler's last paper, completed just before his death, [221].

In a letter to me dated 24 February, 1988—which I received after hearing of his death—Mahler set me the following problem:

> Let $f(x)$ be a polynomial in x with integral coefficients which is positive for positive x. Study the integers x for which the representation of $f(x)$ to the base $g \geq 3$ has only digits 0 and 1.

Concerning his life's work he also happened to write, in that letter: "When my old papers first appeared, they produced little interest in the mathematical world, and it was only in recent times that they have been rediscovered and found useful … ". That grossly underrates the impact of his work in the past, but correctly notices the richness of even his minor remarks.

2. Mahler's mathematics

There are a number of interlinked themes that run through Mahler's work. The primary one, and in any case the one on which I feel best able to comment, and therefore will concentrate, is diophantine approximation and transcendence theory. Even then, I emphasise those parts of his work that most influence my thinking.

[1]Once at an ARGS interview I was told that "one of the referees" had suggested that I might be better off with a TI than an HP 'computer' as requested.

[2]On at least two occasions—when I learned to interpolate p-adically and avoid the Schnirelman integral; and when I saw how to reduce the dependence on p in Baker's inequality from p^n to $p(\log p)^c$—this turned out to be very good advice.

I recall Mahler saying disappointedly that he had never proved a 'major result', that his main contribution had been to prove mere lemmas. In that sense his 'near proof' [169] of the transcendence of Euler's constant γ must have been a major disappointment. Yet even there he proves such results as the transcendence of

$$\frac{\pi Y_0(2)}{2J_0(2)} - \gamma \ ;$$

γ does not separate from the Bessel functions because of a nonlinear algebraic relation relating the Bessel functions appearing in the auxiliary function featuring in the proof.

The future will tell which of his papers have been the most inappropriately neglected. My recollection is that on a number of occasions Mahler mentioned [43] (and [152]) on periodic algorithms as well warranting further study. He was disappointed by the apparent lack of response to his paper [157] on ideal bases.

Mahler's Method. Of course Mahler introduced entire new subjects: Chronologically, the first is 'Mahler's method', so named by Loxton and the author (1977) which is introduced in Mahler's papers [4,7,8] but was then long-neglected, except for Mahler's foray into Chinese [96] and its translation [115], until revived by him in [170]. At that time Mahler suggested that I might like to apply more recent results in diophantine approximation to settle some of the outstanding problems. Eventually John Loxton and I took up that challenge and helped to establish the existence of a new method (in fact, by then a rather old method) in transcendence theory. The method yields transcendence and algebraic independence results for the values at algebraic points of 'Mahler functions', to wit power series f satisfying functional equations with simplest example the so-called Fredholm series

$$f(z) = \sum_{h=0}^{\infty} z^{2^h} \text{ with } f(z^2) = f(z) - z \ ;$$

multivariable examples include the remarkable series

$$F_\omega(z_1 , z_2) = \sum_{h_1=1}^{\infty} \sum_{1 \le h_2 \le h_1\omega} z_1^{h_1} z_2^{h_2}$$

which satisfy a chain of functional equations

$$F_{\omega_k}(z_1^{a_k} z_2 , z_1) = -F_{\omega_{k-1}}(z_1 , z_2) + \frac{z_1^{a_k+1} z_2}{(1 - z_1^{a_k} z_2)(1 - z_1)} \ ;$$

here $0 < \omega < 1$ and ω has the continued fraction expansion

$$\omega = [0 \ , a_1 \ , a_2 \ , \dots \ , a_{k-1} \ , \omega_{k-1}] \qquad k = 1 , 2 , \dots \ .$$

https://doi.org/10.1017/S1446788700034558 Published online by Cambridge University Press

When ω is a quadratic irrational the periodicity of the continued fraction expansion allows one to compact the chain of functional equations to a single functional equation, yielding the celebrated result that

$$\sum_{h=1}^{\infty} \lfloor h\alpha \rfloor z^h$$

is transcendental for all quadratic irrational α and algebraic z satisfying $0 < z < 1$. Generally, following suggestions of Mahler, Loxton and the author (1977a) eventually showed that, for example, viewed as a function of ω the function $F_\omega(\frac{1}{2})$ is rational for rational ω and transcendental for all irrational ω.

In an unexpected *tour de force* Masser (1982) provided a major breakthrough in the several variable theory by applying Chabauty's method to complete the work initiated by Mahler in [7]. Further related papers of Mahler include [195,199,204]. Mahler dearly wished to confirm a connection of this work with his work on modular functions, cf [182,188]; see also [201,208]. Values of Mahler functions have yielded examples of numbers with interesting transcendence type; contributors include P. Morton, Kumiko Nishioka, P.-G. Becker and N. Wass.

Mahler's method took on a new significance when Mendès France and his co-workers (1980) observed that some of Mahler's examples arose in their study of finite automata. This connection between finite automata and functional equations was known some years earlier to Cobham (1968), but was apparently never published. The survey 'Folds!' shows the ubiquity of the notions of finite automaton and automatic sequence. The transcendence of the values of Mahler functions reappears as a theorem that the sequence of digits of the decimal representation of an irrational algebraic number never form an automatic sequence. Finite automata do relate to the mathematical mainstream: the (good) reductions of diagonals of rational functions in several variables—this includes all power series representing algebraic functions—have a natural reinterpretation as Mahler functions; see my survey with Lipshitz. Amusingly, it seems clear that Mahler's interest in 'Mahler functions' arose from the example he found in [1] to comment on Wiener's work, and thus had its genesis in the context of automata.

Mahler's Classification. In his paper [11,13] Mahler introduces his celebrated classification of real and complex numbers into A—the algebraic numbers, and S, T and U numbers; the point being that transcendental numbers in two different classes are algebraically independent. He had already shown in [9] that e is algebraically independent of the Liouville numbers. The existence of T numbers was first shown by Schmidt (1968). Mahler showed [12] that almost all real and almost all complex numbers are

S-numbers and conjectured that in the subclassification the parameter almost always has the one value; this was proved by Sprindzuk (1965). A different but closely related classification is given by Koksma (1939). In [27] Mahler gives the p-adic analogue of his classification.

Perfect Systems. In order to apply his classification, Mahler provides some sharp transcendence measures [11,13]. (The textbook from which I taught myself transcendence theory, that of LeVeque, has a chapter which is essentially a translation of [11]). Mahler achieves that by returning to and generalising the formulae that allowed Hermite to prove the transcendence of e and π and in effect gives a very explicit example applying the transcendence method developed by Siegel. The Padé approximations of [10] foreshadow present important work on effective approximation of algebraic numbers.

The underlying principle [168] is the following: Let $f_1, \dots, f_m$ be power series linearly independent over $\mathbb{C}(z)$. Then for every choice of $\rho_1, \dots, \rho_m$ non-negative integers with sum σ, there are (by elementary linear algebra) polynomials $a_1, \dots, a_m$, not all zero, of respective degrees not exceeding $\rho_1 - 1, \dots, \rho_m - 1$ so that the linear form

$$a_1(z)f_1(z) + a_2(z)f_2(z) + \cdots + a_m(z)f_m(z)$$

has a zero of order $\sigma - 1$ at $z = 0$. If $\alpha_1, \dots, \alpha_m$ are distinct complex numbers then

$$a_1(z)e^{\alpha_1 z} + a_2(z)e^{\alpha_2 z} \cdots + a_m(z)e^{\alpha_m z}$$

has a zero of order *at most* $\sigma - 1$ at $z = 0$. In Mahler's felicitous terminology [168] the vector $e^{\alpha_1 z}, e^{\alpha_2 z}, \dots, e^{\alpha_m z}$ is a *perfect system* of functions. It is now a relatively easy matter to construct m linearly independent polynomial 'approximations' to the vector. Eventually, Shidlovskii shows—this is the essence of the celebrated Shidlovskii's lemma—that if $f_1, \dots, f_m$ are the solutions of a linear differential equation with rational function coefficients then there is a *defect* $\delta = \delta(f)$, *independent* of the ρ_i, so that in any case the order of the zero at $z = 0$ of the linear form does not exceed $\sigma - 1 + \delta$. Mahler's manuscript [168], though written in Groningen in the mid-thirties, was not published until 1968. In the meantime his handwritten manuscript had been the basis of work of Jager in Amsterdam, of the honours essay of Coates and an important part of my doctoral thesis; see also Baker (1966). Other instances of exact constructions include [119] and [164].

Mahler's celebrated inequality: for integers p, $q > 0$

$$|\pi - p/q| > q^{-42}$$

is proved in [119]. The index 42 was subsequently decreased by various authors employing minor refinements of the method. It was a real surprise when Apéry showed by an elementary method that

$$|\pi^2 - p/q| > q^{-11.851\ldots};$$

see my note (1979).

Geometry of numbers. The term 'Geometry of Numbers' was first used by Minkowski to describe arguments based on considerations of packing and covering. In simple situations this leads to striking proofs but it required Mahler's compactness theorem of 1946 to systematise and simplify the intuitive considerations of the subject. Mahler explores the consequences of his compactness theorem in [82,83,84,87,88]. A recent important result in which Mahler's compactness theorem plays an essential role is that of Margulis (1987).

In [79] Mahler proved, independently and almost simultaneously with Hlawka, a form of the Minkowski-Hlawka theorem. In [126,127,128,129] Mahler introduces the notion of the p th compound of a convex body. At first this was considered a useless, if interesting, abstraction. It turned out to be a vital tool in the generalisation by Schmidt (1970) of Roth's theorem. The special case of polar convex bodies [57] serves for the case $n = 3$ of Schmidt's theorem and is applied by Davenport to his study of indefinite quadratic forms; see also Schmidt (1977). Mahler's work [44,53,56] and particularly [57] on Khintchine's transference theorems systematises the observed relationships and shows their natural setting to be convex bodies, their distance functions and their duals.

Polynomials. Several of Mahler's papers (for example [143,144,148,150, 153,154,156]) are concerned with measures for the size of a polynomial

$$f(x) = a_0X^n + \cdots + a_n = a_0(X - \alpha_1)\cdots(X - \alpha_n)$$

and thence with measures for the numbers α_i defined by f. A primary concern is the study of inequalities, important in transcendence theory, between such quantities as the classical height

$$\max_j |a_j|,$$

the length of f

$$L(f) = |a_0| + \cdots + |a_n|,$$

and the size (spoken of as their 'house' by Mahler) of its zeros

$$\overline{|\alpha|} = \max_i |\alpha_i|.$$

To that end Mahler introduces [143] the measure, now known as Mahler's measure,

$$\mathrm{M}(f) = |a_0| \prod_i \max(1, |\alpha_i|) = \exp \int_0^1 \log|f(e^{2\pi it})|\,dt.$$

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One has, congenially,

$$M(fg) = M(f)\,M(g).$$

Nowadays, if $f \in \mathbb{Z}[X]$ is irreducible (and $a_0, \ldots, a_n$ are relatively prime) one defines the absolute logarithmic height $h(\alpha) = \log H(\alpha)$ of a zero α of f by

$$(\deg f)\,h(\alpha) = \log M(f) = \sum_v \max(0, \log|\alpha|_v),$$

where v runs over the appropriately normalised absolute values of the number field $\mathbb{Q}(\alpha)$. This definition is efficient inter alia because it provides an equitable treatment of all absolute values quite eliminating the mystique once possessed by p-adic generalisations of classical diophantine results. It is a measure of the conservatism of transcendence theory that the absolute height is still not in universal use.

Mahler's measure probably first appears in work of Landau (1905). It is central to Lehmer's celebrated question : Suppose $f(X) \in \mathbb{Z}[X]$. Then, by a theorem of Kronecker, $M(f) = 1$ if and only if f is a cyclotomic polynomial. Is $M(f)$ uniformly bounded away from 1 if f is not cyclotomic? The best result known is Dobrowolski's inequality

$$\log M(f) \gg \left(\frac{\log\log\deg f}{\log\deg f}\right)^3.$$

David Boyd's work, see for example his survey (1981), is a major contribution towards this important problem.

p-Adic numbers and p-adic Diophantine Approximation. The p-adic numbers had been introduced by Hensel at the beginning of the century and Hasse had made vital use of them in his study of quadratic forms in the early twenties. Mahler's work helped p-adic numbers to become part of the general mathematical culture.

Suppose

$$\begin{aligned} f(X, Y) &= a_0X^n + a_1X^{n-1}Y + \cdots + a_{n-1}XY^{n-1} + a_nY^n \\ &= a_0(X - \alpha_1Y)\cdots(X - \alpha_nY) \end{aligned}$$

is a binary form of degree $n \geq 3$ defined over $\mathbb{Z}$ and irreducible in $\mathbb{Z}[X, Y]$. Then the diophantine equation $f(X, Y) = m$, m some given integer, has only finitely many solutions $(X, Y) \in \mathbb{Z}^2$. This is essentially because a solution entails that some factor $X - \alpha Y$ is small:

$$|X - \alpha Y| \ll_f |m|/Y^{n-1},$$

and that is impossible for infinitely many Y by Thue's theorem on the approximation of algebraic numbers by rationals.

Mahler's p-adic generalisation (in fact of Siegel's sharpening of Thue's result) allows one to replace the given m by an integer composed from the primes of some given finite set S; alternatively X, Y may be S-integers: they are allowed denominators whose factors come from S. This entails results on the greatest prime factor of integers represented by binary forms [17,20]; or that there are only finitely many S-integral points on a curve of genus 1 [21]. One refers to the generalised equation as the Thue-Mahler equation. Indeed, generally, the suffix '-Mahler' signals the generalisation of a diophantine problem to one in S-integers. In the present spirit, Mahler and Lewis [147] provide bounds for the number of solutions $(X, Y) \in \mathbb{Z}^2$ of $f(X, Y) = m$ as m varies in terms of the number of prime factors of m. This problem has been studied more recently by Bombieri and Schmidt (1987).

In an interesting application, Mahler [135] employs a 2-adic argument to obtain a lower bound for the fractional part of $(3/2)^k$—a result settling the value of $g(k)$ in Waring's problem. However, Mahler's solution is ineffective in k. More recent effective arguments, applying p-adic versions of Baker's inequalities are too weak to apply to Waring's problem.

In a more unexpected way, Mahler's arguments led to the following amusing result [46]: Suppose f is a non-constant polynomial taking integer values at the nonnegative integers. Then the concatenated decimal

$$\phi = 0.f(1)f(2)f(3)\ldots$$

is transcendental. In particular Champernowne's normal number

$$0.123\ldots910111213\ldots$$

is transcendental. Mahler's argument relies on the observation that one readily obtains rational approximations to ϕ with denominators high powers of the base 10, thus composed of the primes 2 and 5 alone. Perhaps disappointingly, Roth's definitive form of the Thue-Siegel inequalities permits a more immediate argument obviating the need for an appeal to the p-adic results.

Mahler initiated the now burgeoning study of transcendence properties of p-adic analytic functions with his paper [14] on the p-adic exponential function and his proof [30] of the p-adic analogue of the Gelfond-Schneider theorem on the transcendence of α^β for algebraic $\alpha \neq 0, 1$ and irrational algebraic β.

In 1933 Skolem deduced results about the number of solutions of certain diophantine problems from a new kind of series expansion. Mahler concluded that in essence the method was p-adic and applied similar techniques to prove the beautiful result [25,31] that the zero Taylor coefficients of a rational function defined over an algebraic number field occur periodically (from some point on). Eventually Lech and independently Mahler [131,131a] generalised the result to arbitrary fields of definition of characteristic zero. Cassels (1976) gives an elegant simplified treatment. For results placing the Lech-Mahler theorem in context see the author's survey (1989).

In [139,145] Mahler proves a criterion for a function defined on the positive integers to have an interpolation continuing it to a continuous function on the p-adic integers. This result is basic for the modern theory of p-adic L-functions. Mahler's book [205] (an extended second edition of [184]) studies the elementary analysis of p-adic functions defined in this way.

The paper [17] includes a proof that if S_0, S_1, S_2 are disjoint nonempty finite sets of rational primes then an equation

$$z_0 + z_1 + z_2 = 0,$$

where z_i is only divisible by primes in S_i, has only finitely many solutions in integers z_0, z_1, z_2. This is the genesis of the important and presently fashionable study of S-unit equations initiated by remarks of the author and Schlickewei, and of Evertse. Dubois and Rhin, and Schlickewei had applied p-adic generalisations of Schmidt's subspace theorem to extend Mahler's result to general S-unit equations

$$z_0 + z_1 + \cdots + z_n = 0,$$

settling a conjecture of Mahler. The more recent results eliminate the requirement that the sets of primes be pairwise disjoint by asking for primitive solutions (in an appropriate sense) of the S-unit equation. There are numerous applications; see the survey of Evertse, Győry, Stewart and Tijdeman.

Mahler regarded p-adic numbers as a special case of g-adic numbers defined by a pseudo-valuation [34,37,38,39]. Here g is any positive integer and, it turns out, the g-adic completion is the product of the p-adic completions over the primes p dividing g.

3. Acknowledgements

I am indebted to John Mack for his major contribution in helping to compile the accompanying list of Mahler's papers and to the many corrections kindly provided by Ian Cassels. I have also borrowed some observations from Cassels' draft remarks on Mahler's mathematics. Various colleagues at the Oberwolfach meeting on Analytische Zahlentheorie and at Macquarie University assisted me with final corrections and additions.

References

Alan Baker (1966), 'A note on the Padé table' *K. Nederl. Akad. Wetensch. Proc. Ser. A* **69**, 596–601

E. Bombieri and W. M. Schmidt (1987), 'On Thue's equation' *Invent. Math.* **88**, 69–81

David W. Boyd (1981), 'Speculations concerning the range of Mahler's measure', *Canad. Math. Bull.* **24**, 453–469

J. W. S. Cassels (1976), 'An embedding theorem for fields', *Bull. Austral. Math. Soc.* **14**, 193–198; Addendum: *ibid* **14**, 479–480

John Coates (1966), 'On the algebraic approximation of functions I, II, III', *K. Nederl. Akad. Wetensch. Proc. Ser. A* **69**, 421–461; IV, *ibid* **70** (1967), 205–212

G. Christol, T. Kamae, M. Mendès France and G. Rauzy (1980), 'Suites algébriques, automates et substitutions', *Bull. Soc. Math. France* **108**, 401–419

A. Cobham (1968), 'On the Hartmanis-Stearns problem for a class of tag machines', Technical report **RC 2178**, IBM Research Centre, Yorktown Heights, New York, 15pp

Michel Dekking, Michel Mendès France and Alf van der Poorten (1982), 'FOLDS!', *The Mathematical Intelligencer* **4**, 130–138; II: 'Symmetry disturbed', *ibid.* 173–181; III: 'More morphisms', *ibid.* 190–195

H. Davenport (1958), 'Indefinite quadratic forms in many variables (II)', *Proc. London Math. Soc.* **8**, 109–126

E. Dobrowolski (1979), 'On a question of Lehmer and the number of irreducible factors of a polynomial', *Acta Arith.* **34**, 391–401

E. Dubois and G. Rhin (1976), 'Sur la majoration de formes linéaires à coefficients algébriques réels et p-adiques (Démonstration d'une conjecture de K. Mahler)', *C. R. Acad. Sc. Paris* **A282**, 1211

Jan-Hendrik Evertse (1984), 'On sums of S-units and linear recurrences', *Compositio Math.* **53**, 225–244

J.-H. Evertse, K. Győry, C. L. Stewart and R. Tijdeman (1988), 'S-unit equations and their applications', in *New advances in transcendence theory* ed. Alan Baker, (Durham Symposium on Transcendental Number Theory 1986), Cambridge Univ. Press, 110–174

C. Hermite (1873), 'Sur la fonction exponentielle, *Oeuvres*, t. III, 151–181

C. Hermite (1893), 'Sur la généralization des fractions continues algébriques', *Oeuvres*, t. IV, 357–377

H. Jager (1964), 'A multidimensional generalization of the Padé table', *Proc. K. Nederl. Akad. v. Wetenschappen Series A* **67**, 192–249

A. Khintchine (1926), 'Über eine Klasse linearer Diophantischer Approximationen', *Rend. Circ. Mat. Palermo* **50**, 170–195

J. F. Koksma (1939), 'Über die Mahlersche Klassenteilung der transzendenten Zahlen und die Approximation komplexer Zahlen durch algebraische Zahlen', *Monatsh. Math. Phys.* **48**, 176–189

E. Landau (1905), 'Sur quelques théorèmes de M. Petrovich relatifs aux zéros des fonctions analytiques', *Bull. Soc. Math. France* **33**, 251–261

Christer Lech (1953), 'A note on recurring series', *Ark. Mat.* **2**, 417–421

D. H. Lehmer (1933), 'Factorization of certain cyclotomic functions', *Ann. Math.* **34**, 461–479

W. J. LeVeque (1961), *Topics in Number Theory* Vol. **2**, Addison-Wesley

L. Lipshitz and A. J. van der Poorten (1990), 'Rational functions, diagonals, automata and arithmetic', in *Number Theory*, ed. Richard A. Mollin, (First Conference of the Canadian Number Theory Association, Banff 1988) Walter de Gruyter Berlin · New York, 339–358

J. H. Loxton and A. J. van der Poorten (1977a), 'Arithmetic properties of certain functions in several variables III', *Bull. Austral. Math. Soc.* **16**, 15–47

J. H. Loxton and A. J. van der Poorten (1977b), 'Transcendence and algebraic independence by a method of Mahler', in *Transcendence theory—advances and applications*, ed. A. Baker and D. W. Masser, Academic Press London and New York, Chapter 15, 211–226

J. H. Loxton and A. J. van der Poorten (1988), 'Arithmetic properties of automata: regular sequences', *J. für Math.* **392**, 57–69

G. A. Margulis (1987), 'Formes quadratiques indéfinies et flots unipotents sur les espaces homogènes' *Comptes Rendus Acad. Sci. Paris* **304**, 249–253

D. W. Masser (1982), 'A vanishing theorem for power series', *Invent. Math.* **67**, 275–296

M. Mendès France (1980), 'Nombres algébriques et théorie des automates', *l'Ens. Math.* (2) **26**, 193–199

Alfred van der Poorten (1979), 'A proof that Euler missed ... Apéry's proof of the irrationality of $\zeta(3)$; An informal report', *The Mathematical Intelligencer* **1**, 195-203

A. J. van der Poorten (1989), 'Some facts that should be better known; especially about rational functions', in *Number Theory and Applications*, ed. Richard A.Mollin, (NATO - Advanced Study Institute, Banff, 1988) Kluwer Academic Publishers Dordrecht, 497-528

A. J. van der Poorten and H. P. Schlickewei (1991), 'Additive relations in number fields', *J. Austral. Math. Soc.*

H. P. Schlickewei (1977a), 'The $\mathfrak{p}$-adic Thue-Siegel-Roth-Schmidt theorem', *Arch. Mat.* **29**, 267–270

H. P. Schlickewei (1977b), 'Über die diophantische Gleichung $x_1 + x_2 + \cdots + x_n = 0$ ', *Acta Arith.* **33**, 183–185

W. M. Schmidt (1968), ' T-number do exist', *Symposia Mathematica* **IV** INDAM, Rome Academic Press, London; see also 'Mahler's T-numbers' *Proc. Symposia in Pure Math.* (Stonybrook 1969) **XX** Amer. Math. Soc., (1971), 275–286

W. M. Schmidt (1970), 'Simultaneous approximation to algebraic numbers by rationals' *Acta Math.* **125**, 189–201

W. M. Schmidt (1977), *Small fractional parts of polynomials*, CBMS Regional Conf. Ser. **32** Amer. Math. Soc., 41pp

A. B. Shidlovskii (1959), 'Transcendality and algebraic independence of the values of certain functions', *Amer. Math. Soc. Transl.* (2) **27** (1963), 191–230 = *Trudy Moscov. Mat. Obsc.* **8**, 283–320; 'A criterion for algebraic independence of the values of a class of entire functions', *Amer. Math. Soc. Transl.* (2) **22** (1962), 339–370 = *Izv. Akad. Nauk SSSR Ser. Mat.* **23** (1959), 35–66 and see further references in [200]

C. L. Siegel (1929), 'Über einige Anwendungen diophantischer Approximationen', *Preuß. Akad. Wiss. Phys.-mat. Kl. Berlin* No.1 = *Gesammelte Abhandlungen* **I** Springer Verlag (1966), 209–266

V. G. Sprindzuk (1965), 'A proof of Mahler's conjecture on the measure of the set of S-numbers', *Izv. Akad. Nauk SSSR (ser. mat.)* **29**, 379-436

Appendix 1

Mahler's Publications

[1] On the translation properties of a simple class of arithmetical functions (part two of: Norbert Wiener, The spectrum of an array and its application to the study of the translation properties of a simple class of arithmetical functions), *J. Math. and Physics* **6**, (1927), 158–163 **[Jbuch 53, 205]** Remark: Reprinted in *Publ. MIT Ser. II* **62** No. 118 (1927)

[2] Über die Nullstellen der unvollständigen Gammafunktionen, Dr. Dissertation Frankfurt, = *Rendiconti del Circolo Matematico di Palermo* **54** (1930), (1927), 1–41 **[Jbuch 56, 310]** Remark: Note "Lebenslauf" at end of thesis.

[3] Über einen Satz von Mellin, *Math. Ann.* **100**, (1928), 384–398 **[Jbuch 54, 369]**

[4] Arithmetische Eigenschaften der Lösungen einer Klasse von Funktionalgleichungen, *Math. Ann.* **101**, (1929), 342–366 **[Jbuch 55, 115; 56, 185]** Remark: Plus "Berichtigung", *Math. Ann.* **103**

[5] Über die Nullstellen der Abschnitte der hypergeometrischen Reihe , *Math. Ann.* **101**, (1929), 367–374 **[Jbuch 55, 219]**

[6] Zur Fortsetzbarkeit gewisser Dirichletscher Reihen, *Math. Ann.* **102**, (1929), 30–48 **[Jbuch 55, 201]**

[7] Über das Verschwinden von Potenzreihen mehrerer Veränderlichen in speziellen Punktfolgen, *Math. Ann.* **103**, (1930), 573–587 **[Jbuch 56, 186]**

[8] Arithmetische Eigenschaften einer Klasse transzendental-transzendente Funktionen, *Math. Z.* **32**, (1930), 545–585 **[Jbuch 55, 186]**

[9] Über Beziehungen zwischen der Zahl e und Liouvilleschen Zahlen, *Math. Z.* **31**, (1930), 729–732 **[Jbuch 56, 188]**

[10] Ein Beweis des Thue-Siegelschen Satzes über die Approximation algebraischer Zahlen für binomische Gleichungen, *Math. Ann.* **105**, (1931), 267–276 **[Zblatt 2, 184]**

[11] Zur Approximation der Exponentialfunktion und des Logarithmus. Teil I, *J. für Math.* **166**, (1931), 118–136 **[Zblatt 3, 151]**

[12] Über das Mass der Menge aller S-Zahlen, *Math. Ann.* **106**, (1932), 131–139 **[Zblatt 3, 246]**

[13] Zur Approximation der Exponentialfunktion und des Logarithmus. Teil II, *J. für Math.* **166**, (1932), 137–150 **[Zblatt 3, 388]**

[14] Ein Beweis der Transzendenz der P-adischen Exponentialfunktion, *J. für Math.* **169**, (1932), 61–66 **[Zblatt 6, 11]**

[15] Einige Sätze über Diophantische Approximationen, *Jahresbericht d. Deutschen Math. Verein.* **41**, (1932), 74–76

[16] Über die Darstellung von Zahlen durch Binärformen höheren Grades, Kongressbericht, Zürich, 1932, (1932), 2pp

[17] Zur Approximation algebraischer Zahlen. I (Über den grössten Primteiler binärer Formen), *Math. Ann.* **107**, (1933), 691–730 **[Zblatt 6, 105]**

[18] Zur Approximation algebraischer Zahlen II (Über die Anzahl der Darstellungen ganzer Zahlen durch Binärformen), *Math. Ann.* **108**, (1933), 37–55 **[Zblatt 6, 156]**

[19] Zur Approximation algebraischer Zahlen III (Über die mittlere Anzahl der Darstellungen grosser Zahlen durch binäre Formen), *Acta Math.* **62**, (1933), 91–166 **[Zblatt 8, 198]**

[20] Über den grössten Primteiler der Polynome $X^2 \mp 1$, *Archiv Math. og. Naturv.* **41**, (1933), 1–8 **[Zblatt 12, 394]**

[21] Über die rationalen Punkte auf Kurven vom Geschlecht Eins, *J. für Math.* **170**, (1933), 168–178 **[Zblatt 8, 200]**

[22] Zur Approximation P-adischer Irrationalzahlen, *Nieuw Arch. Wisk.* **18**, (1934), 22–34 **[Zblatt 9, 200]**

https://doi.org/10.1017/S1446788700034558 Published online by Cambridge University Press

[23] Über die Darstellungen einer Zahl als Summe von drei Biquadraten, *Mathematica (Zutphen)* 3, (1934), 69–72 **[Zblatt 9, 298]**
[24] Über Diophantische Approximationen in Gebiete der p-adische Zahlen, *Jahresbericht d. Deutschen Math. Verein.* **44**, (1934), 250–255 **[Zblatt 10, 198]**
[25] Eine arithmetische Eigenschaft der recurrierenden Reihen, *Mathematica (Zutphen)* **3**, (1934), 153–156 **[Zblatt 10, 390]**
[26] On Hecke's theorem on the real zeros of the L-functions and the class number of quadratic fields, *J. London Math. Soc.* **9**, (1935), 298–302 **[Zblatt 10, 250]**
[27] Über eine Klasseneinteilung der P-adischen Zahlen, *Mathematica (Zutphen)* **3B**, (1935), 177–185 **[Zblatt 11, 58]**
[28] On the lattice points on curves of genus 1, *Proc. London Math. Soc.* **39**, (1935), 431–466 **[Zblatt 12, 150]**
[29] On the division-values of Weierstrass's $\wp$-function, *Quart. J. Math. Oxford* **6**, (1935), 74–77 **[Zblatt 11, 117]**
[30] Über transzendente P-adische Zahlen, *Compositio Math.* **2**, (1935), 259–275; A Correction, *ibid.* (1948), 2pp **[Zblatt 12, 53]**
[31] Eine arithmetische Eigenschsaft der Taylor-Koeffizienten rationaler Funktionen, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **38**, (1935), 50–60 **[Zblatt 10, 390]**
[32] Über den grössten Primteiler spezieller Polynome zweiten Grades, *Archiv Math. og. Naturv.* **41, Nr 6**, (1935), 3–26 **[Zblatt 13, 389]**
[33] (with J. Popken) Ein neues Prinzip für Transzendenzbeweise, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **38**, (1935), 864–871 **[Zblatt 12, 341]**
[34] Über Pseudobewertungen, I, *Acta Math.* **66**, (1935), 79–119 **[Zblatt 13, 51]**
[35] Eine arithmetische Eigenschaft der kubischen Binärformen, *Nieuw Arch. Wisk.* **18**, (1936), 1–9 **[Zblatt 14, 8]**
[36] Ueber Polygone mit Um- oder Inkreis, *Mathematica (Zutphen)* **4A**, (1936), 33–42 **[Zblatt 13, 176]**
[37] Über Pseudobewertungen II, *Acta Math.* **67**, (1936), 51–80 **[Zblatt 14, 340]**
[38] Über Pseudobewertungen III, *Acta Math.* **67**, (1936), 283–328 **[Zblatt 16, 5]**
[39] Über Pseudobewertungen Ia, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **39**, (1936), 57–65 **[Zblatt 13, 147]**
[40] Note on Hypothesis K of Hardy and Littlewood, *J. London Math. Soc.* **11**, (1936), 136–138 **[Zblatt 13, 391]**
[41] Ein Analog zu einem Schneiderschen Satz, I, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **39**, (1936), 633–640 **[Zblatt 14, 205]**
[42] Ein Analog zu einem Schneiderschen Satz, II, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **39**, (1936), 729–737 **[Zblatt 14, 205]**
[42a] Pseudobewertungen, *Proc. ICM, Oslo* (1936), 1p.
[43] Über die Annäherung algebraischer Zahlen durch periodische Algorithmen, *Acta Math.* **68**, (1937), 109–144 **[Zblatt 17, 57]**
[44] Neuer Beweis eines Satzes von A. Khintchine, *Mat. Sb.* **I, 43**, (1937), 961–963 **[Zblatt 16, 155]**
[45] Über die Dezimalbruchentwicklung gewisser Irrationalzahlen, *Mathematica (Zutphen)* **B6**, (1937), 22–26 **[Zblatt 18, 111]**
[46] Arithmetische Eigenschaften einer Klasse von Dezimalbrüchen, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **40**, (1937), 421–428 **[Zblatt 17, 56]**
[47] (with P. Erdős) On the number of integers which can be represented by a binary form, *J. London Math. Soc.* **13**, (1938), 134–139 **[Zblatt 18, 344]**
[48] On a special class of Diophantine equations, I, *J. London Math. Soc.* **13**, (1938), 169–173 **[Zblatt 19, 250]**
[49] On a special class of Diophantine equations, II, *J. London Math. Soc.* **13**, (1938), 173–177 **[Zblatt 19, 250]**

[50] Ein *P*-adisches Analogon zu einem Satz von Tchebycheff, *Mathematica (Zutphen)* **B7**, (1938), 2–6 **[Zblatt 18, 346]**
[51] On the fractional parts of the powers of a rational number, *Acta Arith.* **3**, (1938), 89–93 **[Zblatt 19, 250]**
[52] Über einen Satz von Th. Schneider, *Acta Arith.* **3**, (1938), 94–101 **[Zblatt 19, 250]**
[53] A theorem on inhomogenous Diophantine inequalities, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **41**, (1938), 634–637 **[Zblatt 19, 51]**
[54] Eine Bemerkung zum Beweis der Eulerschen Summenformel, *Mathematica (Zutphen)* **B7**, (1938), 33–42 **[Zblatt 19, 403]**
[55] On Minkowski's theory of reduction of positive definite quadratic forms, *Quart. J. Math. Oxford* **9**, (1938), 259–262 **[Zblatt 19, 395]**
[56] Ein Übertragungsprinzip für lineare Ungleichungen, *Math. Časopis* **68**, (1939), 85–92 **[Math. Rev. 1, 202]**, **[RNT H05-1]**
[57] Ein Übertragungsprinzip für konvexe Körper, *Math. Časopis* **68**, (1939), 93–102 **[Math. Rev. 1, 202H05-2]**
[58] (with P. Erdős) Some arithmetical properties of the convergents of a continued fraction, *J. London Math. Soc.* **14**, (1939), 12–18 **[Zblatt 20, 390]**
[59] Ein Minimalproblem für convexe Polygone, *Mathematica (Zutphen)* **B7**, (1938), 118–127 **[Zblatt 20, 50]**
[60] On a the solutions of algebraic differential equations, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **42**, (1939), 61–63 **[Zblatt 21, 27]**
[61] A proof of Hurwitz's theorem, *Mathematica (Zutphen)* **8B**, (1939), 57–61 **[Math. Rev. 1, 39]**, **[RNT J28-1]**
[62] Bemerkungen über die Diophantischen Eigenschaften der reellen Zahlen, *Mathematica (Zutphen)* **8B**, (1939), 11–16 **[Zblatt 21, 296]**
[63] On the minimum of positive definite Hermitian forms, *J. London Math. Soc.* **14**, (1939), 137–143 **[Zblatt 22, 6]**
[64] On a geometrical representation of p-adic numbers, *Ann. of Math.* **(2) 41**, (1940), 8–56 **[Math. Rev. 1, 295]**, **[RNT J64-2]**
[65] On the product of two complex linear plynomials in two variables, *J. London Math. Soc.* **15**, (1940), 213–236 **[Math. Rev. 2, 148]**, **[RNT J56-1]** Remark: Mahler writes "This is an extension of an earlier paper which was accepted for publication by Acta Arith. in February 1939 of which I had just received the first proofs when war broke out".
[66] Note on the sequence $\sqrt{n}$ (mod 1), *Nieuw Arch. Wisk.* **20**, (1940), 176–178 **[Math. Rev. 1, 202]**, **[RNT J24-1]**
[67] (with G. Billing) On exceptional points on cubic curves, *J. London Math. Soc.* **15**, (1940), 32–43 **[Zblatt 26, 199]**, **[Math. Rev. 1, 266]**
[68] On a special functional equation, *J. London Math. Soc.* **15**, (1940), 115–123 **[Math. Rev. 2, 133]**, **[RNT P80-3]**
[69] Über Polynome mit ganzen rationalen Koeffizienten, *Mathematica (Zutphen)* **8B**, (1940), 173–182 **[Math. Rev. 2, 148]**, **[RNT J84-2]**
[70] On reduced positive definite ternary quadratic forms, *J. London Math. Soc.* **15**, (1940), 193–195 **[Zblatt 27, 151]**, **[Math. Rev. 2,119]**
[71] On a property of positive definite ternary quadratic forms, *J. London Math. Soc.* **15**, (1940), 305–320 **[Math. Rev. 2, 25]**, **[RNT J48-1]**
[72] An analogue to Minkowski's Geometry of Numbers in a field of series, *Ann. of Math.* **(2), 42**, (1941), 488–522 **[Math. Rev. 2, 350]**, **[RNT H05-4]**
[73] On ideals in the Cayley-Dickson algebra, *Proc. Royal Irish Academy* **48**, (1942), 123–133 **[Math. Rev. 4, 185]**
[74] Remarks on ternary Diophantine equations, *Amer. Math. Monthly* **49**, (1942), 372–378 **[Math. Rev. 4, 34]**, **[RNT N32-1]**
[75] Note on lattice points in star domains, *J. London Math. Soc.* **17**, (1942), 130–133 **[Math. Rev. 4, 212]**, **[RNT H15-2]**

[76] On lattice points in an infinite star domain, *J. London Math. Soc.* **18**, (1943), 233–238 **[Math. Rev. 6, 11], [RNT H20-4]**
[77] A problem of Diophantine approximation in quaternions, *Proc. London Math. Soc.(2)* **48**, (1944), 435–466 **[Math. Rev. 8, 137], [RNT J32-3]**
[78] (with B. Segre) On the densest packing of circles, *Amer. Math. Monthly* **51**, (1944), 261–270 **[Zblatt 60, 117; 60, 349], [Math. Rev. 6, 16]**
[79] On a theorem of Minkowski on lattice points in non-convex point sets, *J. London Math. Soc.* **19**, (1944), 201–205 **[Math. Rev. 7, 244], [RNT H25-3]**
[80] On lattice points in the domain $|xy| \leq 1$, $|x+y| \leq \sqrt{5}$, and applications to asymptotic formulae in lattice point theory, I, *Proc. Cambridge Phil. Soc.* **40**, (1944), 107–116 **[Math. Rev. 6, 119], [RNT H20-4]**
[81] On Lattice points in the domain $|xy| \leq 1$, $|x+y| \leq \sqrt{5}$, and applications to asymptotic formulae in lattice point theory, II, *Proc. Cambridge Phil. Soc.* **40**, (1944), 116–120 **[Math. Rev. 6, 119], [RNT H20-4]**
[82] A theorem of B. Segre, *Duke Math. J.* **12**, (1945), 367–371 **[Math. Rev. 6, 258], [RNT J08-2]**
[83] Lattice points in two-dimensional star domains, I, *Proc. London Math. Soc.* **49**, (1946), 128–157 **[Math. Rev. 8, 12], [RNT H15-6]**
[84] Lattice points in two-dimensional star domains, II, *Proc. London Math. Soc.* **49**, (1946), 158–167 **[Math. Rev. 8, 195], [RNT H20-10]**
[85] Lattice points in two-dimensional star domains, III, *Proc. London Math. Soc.* **49**, (1946), 168–183 **[Math. Rev. 8, 19], [RNT H20-11]**
[86] On lattice points in a cylinder, *Quart. J. Math. Oxford* **17**, (1946), 16–18 **[Math. Rev. 7, 368], [RNT H10-3]**
[87] On lattice points in n-dimensional star bodies, I, Existence theorems, *Philos. Trans. Roy. Soc. London Ser. A* **187**, (1946), 151–187 **[Math. Rev. 8, 195], [RNT H15-7]**
[88] Lattice points in n-dimensional star bodies. II. Reducibility theorems. I, II, III, IV, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **49**, (1946), 331–343, 444–454, 524–532, 622–631 = *Indag. Math.* **8**, 200–212, 299–309, 343–351, 381–390 **[Math. Rev. 8, 12], [RNT H15-8,9]**
[89] (with H. Davenport) Simultaneous Diophantine approximation, *Duke Math. J.* **13**, (1946), 105–111 **[Math. Rev. 7, 506], [RNT J12-6]**
[90] Lattice points in n-dimensional star bodies, *Rev. Univ. Nac. Tucumán, A* **5**, (1946), 113–124 **[Math. Rev. 8, 566], [RNT H15-10]**
[91] The theorem of Minkowski-Hlawka, *Duke Math. J.* **13**, (1946), 611–621 **[Math. Rev. 8, 444], [RNT H25-5]**
[92] On reduced positive definite quarternary quadratic forms, *Nieuw Arch. Wisk.* (1946), 207–212 **[Math. Rev. 8, 369], [RNT J44-4]**
[93] A remark on the continued fractions of conjugate algebraic numbers, *Simon Stevin* **25**, (1947), 45–48 **[Math. Rev. 8, 318], [RNT A56-10]**
[94] On irreducible convex domains, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **50**, (1947), 98–107 = *Indag. Math.* **9**, 73–82 **[Math. Rev. 8, 445], [RNT H15-13]**
[95] On the area and the densest packing of convex domains, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **50**, (1947), 108–118 = *Indag. Math.* **9**, 83–93 **[Math. Rev. 8, 445], [RNT H15-14]**
[96] On the generating functions of integers with a missing digit [in Chinese], *K'o Hsüeh Science* **29**, (1947), 265–267 **[Math. Rev. 9, 79], [RNT J76-13]**
[97] On the adjoint of a reduced positive definite ternary quadratic form, *Sci. Rec. Academia Sinica* **2**, (1947), 21–31 **[Math. Rev. 9, 270], [RNT E20-18]**
[98] On the minimum determinant and the circumscribed hexagons of a convex domain, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **50**, (1947), 692–703= *Indag. Math.* **9**, 326–337 **[Math. Rev. 9, 10], [RNT H05-10]**

[99] (with K.C. Hallum) On the minimum of a pair of positive definite Hermitean forms, *Nieuw Arch. Wisk.* **22**, (1948), 324–354 **[Math. Rev. 9, 334]**, **[RNT J28-4]**

[100] On the admissible lattices of automorphic star bodies, *Sci. Rec. Academia Sinica* **2**, (1948), 146–148 **[Math. Rev. 1, 102]**, **[RNT H15-22]**

[101] On lattice points in polar reciprocal convex domains, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **51**, (1948), 176–179 = *Indag. Math.* **10**, 482–485 **[Math. Rev. 9, 501]**, **[RNT H05-11]**

[102] Sui determinante minimi delle sezione di un corpo convesso, *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. (8)* **5**, (1948), 251–252 **[Math. Rev. 10, 593]**, **[RNT H05-17]**

[103] On the successive minima of a bounded star domain, *Ann. Mat. Pura Appl. (4)* **27**, (1948), 153–163 **[Math. Rev. 11, 13]**, **[RNT H15-37]**

[104] On the critical lattices of arbitrary point sets, *Canad. J. Math.* **1**, (1949), 78–87 **[Math. Rev. 10, 355]**, **[RNT H15-29]**

[105] On the minimum determinant of a special point set, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **52**, (1949), 633–642= *Indag. Math.* **11** **[Math. Rev. 10, 512]**, **[RNT H15-33]**

[106] On a theorem of Liouville in fields of positive characteristic, *Canad. J. Math.* **1**, (1949), 397–400 **[Math. Rev. 11, 159]**, **[RNT J68-9]**

[107] On Dyson's improvement of the Thue-Siegel theorem, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen* **52**, (1949), 449–458 = *Indag. Math.* **11**, 1175–1184 **[Math. Rev. 11, 583]**, **[RNT J68-9]**

[108] (with W. Ledermann) On lattice points in a convex decagon, *Acta Math. Acad. Sci. Hungar.* **81**, (1949), 319–351 **[Math. Rev. 11, 233]**, **[RNT H10-15]**

[109] On the continued fractions of quadratic and cubic irrationals, *Ann. Mat. Pura Appl. (4)* **30**, (1949), 147–172 **[Math. Rev. 12, 245]**, **[RNT A56-19]**

[109a] A correction to "On the continued fractions of quadratic and cubic irrationals", *Compositio Math.* **8**, (1949), p112 **[Math. Rev. 12, 319]**, **[RNT Q25-5]**

[110] On the arithmetic on algebraic curves, in *Seminar on Algebra and Number Theory*, Chicago, (1949), 28–32

[111] On algebraic relations between two units of an algebraic field, in "Algèbre et théorie des nombres", *Colloques internationaux du CNRS* **24**, (1949), 47–55. CNRS, Paris, 1950 **[Math. Rev. 13, 202]**, **[RNT D40-16]**

[112] On a theorem of Dyson [in Russian], *Mat. Sb.* **26**, (1950), 457–462 **[Math. Rev. 12, 319]**, **[RNT J16-10]**

[113] Geometry of numbers, Lectures given at the University of Colorado in the summer of 1950, (1950),

[114] (with J.W.S. Cassels and W. Ledermann) Farey section in $k(i)$ and $k(\rho)$, *Philos. Trans. Roy. Soc. London Ser. A* **243**, (1951), 585–626 **[Math. Rev. 13, 323]**, **[RNT J56-13]**

[114a] Farey section in the fields of Gauß and Eisenstein, *Proc. International Congress* Cambridge, Mass. 1950 **1**, (1952), 281–285 **[Math. Rev. 13, 538]**

[115] On the generating function of the integers with a missing digit, *J. Indian Math. Soc.* **15A**, (1951), 34–40 **[Math. Rev. 13, 213]**, **[RNT J76-14]** Remark: Translation of (96)

[116] On a question in elementary geometry, *Simon Stevin* **28**, (1951), 90–97 **[Math. Rev. 13, 152]**

[117] On the lattice determinants of two particular point sets, *J. London Math. Soc.* **28**, (1953), 229–232 **[Math. Rev. 14, 850]**, **[RNT H20-19]**

[118] On the approximation of logarithms of algebraic numbers, *Philos. Trans. Roy. Soc. London Ser. A* **245**, (1953), 371–398 **[Math. Rev. 14, 624]**, **[RNT J80-8]**

[119] On the approximation of π, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen Ser. A* **56**, (1953), 30–42 = *Indag. Math.* **15**, 30–42 **[Math. Rev. 14, 957]**, **[RNT J80-9]**

[120] (with J. Popken) Over een Maximumprobleem uit de Rekenkunde, *Nieuw Arch. Wisk. (3)* **1**, (1953), 1–15 **[Math. Rev. 14, 852]**

[121] On the greatest prime factor of $ax^m + by^n$, *Nieuw Arch. Wisk. (3)* **1**, (1953), 113–122 **[Math. Rev. 15, 289]**, **[RNT N32-14]**

[122] (with P. M. Cohn) On the composition of pseudo-valuations, *Nieuw Arch. Wisk. (3)* **1**, (1953), 161–198 **[Math. Rev. 15, 395]**

[123] A problem in elementary geometry, *Math. Gaz.* **37**, (1954), 241–243 **[Math. Rev. 16, 738]**

[124] On a problem in the geometry of numbers, *Rendi. Mat. Appl. (5)* **14**, (1954), 38–41 **[Math. Rev. 16, 802]**, **[RNT H05-38]**

[125] On a problem in Diophantine approximations, *Archiv Math. og. Naturv.* **6**, (1955), 208–214 **[Math. Rev. 16, 1002]**, **[RNT J12-29]**

[126] On compound convex bodies, I, *Proc. London Math. Soc. (3)* **5**, (1955), 358–379 **[Math. Rev. 17, 589]**, **[RNT H05-46]**

[127] On compound convex bodies, II, *Proc. London Math. Soc. (3)* **5**, (1955), 380–384 **[Math. Rev. 17, 589]**, **[RNT H05-47]**

[128] The p th compound of a sphere, *Proc. London Math. Soc. (3)* **5**, (1955), 385–391 **[Math. Rev. 17, 402]**, **[RNT H05-49]**

[129] On the minima of compound quadratic forms, *Czech. Math. J. (5)* **80**, (1955), 180–193 **[Math. Rev. 17, 589]**, **[RNT H05-48]**

[130] A remark on Siegel's theorem on algebraic curves, *Mathematika* **2**, (1955), 116–127 **[Math. Rev. 18, 565]**, **[RNT D40-27]**

[131] On the Taylor coefficients of rational functions, *Proc. Cambridge Phil. Soc.* **52**, (1956), 39–48 **[Math. Rev. 17, 597]**, **[RNT Q05-25]**

[131a] Addendum to "On the Taylor coefficients of rational functions", *Proc. Cambridge Phil. Soc.* **53**, (1957), p544 **[Math. Rev. 19, 691]**, **[RNT Q05-26]**

[132] Invariant matrices and the geometry of numbers, *Proc. Roy. Soc. Edinburgh A* **64**, (1956), 223–238 **[Math. Rev. 18, 196]**, **[RNT H05-50]**

[133] A property of the star domain $|xy| \leq 1$, *Mathematika* **3**, (1956), p80 **[Math. Rev. 18, 21]**, **[RNT H20-21]**

[134] Über die konvexen Körper, die sich einem Sternkörper einbeschrieben lassen, *Math. Z.* **66**, (1957), 25–33 **[Math. Rev. 18, 668]**

[135] On the fractional parts of the powers of a rational number, II, *Mathematika* **4**, (1957), 122–124 **[Math. Rev. 20#3]**, **[RNT J68-19]**

[136] A matrix representation of the primitive residue classes $\pmod{2n}$, *Proc. Amer. Math. Soc.* **8**, (1957), 525–531 **[Math. Rev. 19, 388]**, **[RNT A06-10]**

[137] A factorial series for the rational multiples of e, *Math. Gaz.* **42**, (1958), 13–16 **[Math. Rev. 20#3094]**, **[RNT A68-15]**

[138] On the Chinese remainder theorem, *Math. Nachr.* **18**, (1958), 120–122 **[Math. Rev. 20#3048]**, **[RNT Z15-38]**

[139] An interpolation series for continuous functions of a p-adic variable, *J. für Math.* **199**, (1958), 23–34 **[Math. Rev. 20#]**, **[RNT Q25-11]**

[140] An arithmetic property of groups of linear transformations, *Acta Arith.* **5**, (1959), 197–203 **[Math. Rev. 21#6361]**, **[RNT J28-20]**

[141] On a theorem of Shidlovski, lectures given at the Math. Centrum, Amsterdam, December 1959, (1959),

[142] On a theorem by E. Bombieri, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen Ser. A* **63**, (1960), 245–253 = *Indag. Math.* **22**, 245–253 **[Math. Rev. 22#4705]**, **[RNT J68-24]**

[142a] On a theorem by E. Bombieri, *Proc. Kon. Nederlandsche Akad. v. Wetenschappen Ser. A* **64**, (1960), 141= *Indag. Math.* **23**, 141 **[Math. Rev. 23#A885]**, **[RNT J68-25]**

[143] An application of Jensen's formula to polynomials, *Mathematika* **7**, (1960), 98–100 **[Math. Rev. 23#A1779]**, **[RNT J76-38]**

[144] On the zeros of the derivative of a polynomial, *Philos. Trans. Proc. Roy. Soc. London Ser. A* **264**, (1961), 145–154 **[Math. Rev. 24#A3271]**

https://doi.org/10.1017/S1446788700034558 Published online by Cambridge University Press

[145] A correction the the paper "An interpolation series for continuous functions of a p-adic variable", [Seq. No. 139], *J. für Math.* **208**, (1961), 70–72 **[Math. Rev. 25#3035], [RNT Q25-12]**
[146] *Lectures on Diophantine Approximations I: g-adic Numbers and Roth's Theorem*, Lectures given at the University of Notre Dame in 1957, University of Notre Dame Press, Notre Dame, Indiana, 188 pages, (1961), **[Math. Rev. 26#78], [RNT J02-14]**
[147] (with D.J. Lewis) On the representation of integers by binary forms, *Acta Arith.* **6**, (1961), 333–363 **[Math. Rev. 22#10952], [RNT D40-35]**
[148] On some inequalities for polynomials in several variables, *J. London Math. Soc.* **37**, (1962), 341–344 **[Math. Rev. 25#2036], [RNT J76-40]**
[149] Geometric number theory, Lectures given at the University of Notre Dame in 1962, 151 pp, (1962),
[150] On two extremum properties of polynomials, *Illinois J. Math.* **7**, (1963), 681–701 **[Math. Rev. 28#192]**
[151] On the approximation of algebraic numbers by algebraic integers, *J. Austral. Math. Soc.* **3**, (1963), 408–434 **[Math. Rev. 29#1182], [RNT J68-43]**
[152] Periodic algorithms for algebraic number fields (mimeographed lecture notes), *4th Summer Research Institute of the Austral. Math. Soc.* University of Sydney, January 1964 (1964), 19pp
[153] A remark on a paper of mine on polynomials, *Illinois J. Math.* **8**, (1964), 1–4 **[Math. Rev. 28#2194]**
[154] An inequality for the discriminant of a polynomial, *Michigan Math. J.* **11**, (1964), 257–262 **[Math. Rev. 29#3465], [RNT C05-29]**
[155] Transcendental numbers, *J. Austral. Math. Soc.* **4**, (1964), 393–396 **[Math. Rev. 30#4727], [RNT J02-17]**
[156] An inequality for a pair of polynomials that are relatively prime, *J. Austral. Math. Soc.* **4**, (1964), 418–420 **[Math. Rev. 30#3193], [RNT C05-33]**
[157] Inequalities for ideal bases in algebraic number fields, *J. Austral. Math. Soc.* **4**, (1964), 425–448 **[Math. Rev. 31#1243], [RNT R24-33]**
[158] (with G. Baumschlag) Equations in free metabelian groups, *Michigan Math. J.* **12**, (1965), 417–419 **[Math. Rev. 31#5892]**
[159] Arithmetic properties of lacunary power series with integral coefficients, *J. Austral. Math. Soc.* **5**, (1965), 56–64 **[Math. Rev. 32#7508], [RNT J76-53]**
[160] A remark on recursive sequences, *J. Math. Sci. Delhi* **1**, (1966), 12–17 **[Math. Rev. 35#2853], [RNT B40-52]**
[161] Transcendental numbers, *Encyclopaedia Brittannica*, (1966), 1 column
[162] A remark on Kronecker's theorem, *Enseign Math.* **12**, (1966), 183–189 **[Math. Rev. 35#1550], [RNT J20-39]**
[163] (with G. Szekeres) On the approximation of real numbers by roots of integers, *Acta Arith.* **12**, (1967), 315–320 **[Math. Rev. 35#130]**
[164] Applications of some formulae by Hermite to the approximation of exponentials and logarithms, *Math. Ann.* **168**, (1967), 200–227 **[Math. Rev. 34#5754], [RNT J80-20]**
[165] On a class of entire functions, *Acta Math. Acad. Sci. Hungar.* **18**, (1967), 83–96 **[Math. Rev. 34#5760], [RNT Q05-63]**
[166] On a lemma by A. B. Shidl'ovski [in Russian], *Math. Zametki Acad. Nauk SSR* **2**, (1967), 25–32 **[Math. Rev. 36#2561], [RNT J88-34]**
[167] An unsolved problem on the powers of 3/2, *J. Austral. Math. Soc.* **8**, (1968), 313–321 **[Math. Rev. 37#2694], [RNT K25-29]**
[168] Perfect systems, *Compositio Math.* **19**, (1968), 95–166 **[Math. Rev. 39#458], [RNT J76-64]** Remark: First publication of a manuscript written in the 30s
[169] Applications of a theorem by A. B. Shidlovski, *Philos. Trans. Roy. Soc. London Ser. A* **305**, (1968), 149–173 **[Math. Rev. 37#1322], [RNT J88-37]**

[170] Remarks on a paper by W. Schwarz, *J. Number Theory* **1**, (1969), 512–521 **[Math. Rev. 40#2611], [RNT J76-65]**
[171] On algebraic differential equations satisfied by automorphic functions, *J. Austral. Math. Soc.* **10**, (1969), 445–450 **[Math. Rev. 41#7099]**
[172] Lectures on transcendental numbers (Summer Institute on Number Theory at Stony Brook, 1969), *Proc. Symp. Pure Math.* (Amer. Math. Soc.) **XX**, (1969), 248–274 **[Math. Rev. 47#6620], [RNT I02-200]**
[173] A lecture on the geometry of numbers of convex bodies, *Bull. Amer. Math. Soc.* **77**, (1971), 319–325 **[Math. Rev. 43#1926], [RNT H05-89]**
[174] (with H. Brown) A generalization of Farey sequences, *J. Number Theory* **3**, (1971), 364–370 **[Math. Rev. 44#3959], [RNT B56-14]**
[175] On formal power series as integrals of algebraic differential equations, *Rend. Accad. Naz. dei Lincei* **(8) 50**, (1971), 36–49 **[Math. Rev. 45#8649]**
[176] A remark on algebraic differential equations, *Rend. Accad. Naz. dei Lincei (8)* **50**, (1971), 174–184 **[Math. Rev. 47#3358]**
[177] An arithmetical remark on entire functions, *Bull. Austral. Math. Soc.* **5**, (1971), 191–195 **[Math. Rev. 45#5361]**
[178] An elementary existence theorem for entire functions, *Bull. Austral. Math. Soc.* **5**, (1971), 415–419 **[Math. Rev. 45#515]**
[179] On the order function of a transcendental number, *Acta Arith.* **18**, (1971), 63–76 **[Math. Rev. 45#5090], [RNT I30-203.]**
[180] Transcendental numbers. I. Existence and chief characteristics of transcendental numbers [in Hungarian], *Mat. Lapok* **22**, (1971), 31–50 **[Math. Rev. 58#27822], [RNT I20-224.]**
[181] Transcendental numbers. II. Convergence of Laurent series and formal Laurent series [in Hungarian], *Mat. Lapok* **23**, (1972), 13–23 **[Math. Rev. 58#27823], [RNT I20-224E]**
[182] On the coefficients of the 2^n-th transformation polynomial for $j(\omega)$, *Acta Arith.* **21**, (1972), 89–97 **[Math. Rev. 46#5251], [RNT C10-206]**
[182a] Obituary, L. J. Mordell, *J. Number Theory* **4**, (1972), *iii–iv* **[Math. Rev. 45#5068], [RNT Z20-58G]**
[183] The classification of transcendental numbers, *Proc. Symp. Pure Math.* (Amer. Math. Soc.) **XXIV**, (1973), 175–179 **[Math. Rev. 49#2575], [RNT I30-204]**
[184] *Introduction to p-adic Numbers and their Functions*, Cambridge Tracts in Mathematics **64**, Cambridge University Press, London/New York, (1973), **[Math. Rev. 50#213], [RNT S02-201]**
[185] Arithmetical problems of the digits of the multiples of an irrational number, *Bull. Austral. Math. Soc.* **8**, (1973), 191–203 **[Math. Rev. 47#8448], [RNT K15-204]**
[186] On a class of Diophantine inequalities, *Bull. Austral. Math. Soc.* **8**, (1973), 247–259 **[Math. Rev. 47#8443], [RNT J20-202]**
[187] A p-adic analogue of a theorem by J. Popken, *J. Austral. Math. Soc.* **16**, (1973), 176–184 **[Math. Rev. 49#2578], [RNT I25-203]**
[188] On the coefficients of the transformation polynomials for the modular function, *Bull. Austral. Math. Soc.* **10**, (1974), 197–218 **[Math. Rev. 50#7034], [RNT F05-219]**
[189] On rational approximations of the exponential function at rational points, *Bull. Austral. Math. Soc.* **10**, (1974), 325–335 **[Math. Rev. 50#244], [RNT I25-204]**
[190] Polar analogues of two theorems by Minkowski, *Bull. Austral. Math. Soc.* **11**, (1974), 121–129 **[Math. Rev. 50#7039], [RNT H25-202]**
[191] On the digits of the multiples of an irrational p-adic number, *Proc. Cambridge Phil. Soc.* **76**, (1974), 417–422 **[Math. Rev. 50#221], [RNT K15-206]**
[192] How I became a mathematician, *Amer. Math. Monthly* **81**, (1974), 981–983 **[Math. Rev. 50#1820], [RNT Z20-205]**
[193] On a paper by A. Baker on the approximation of rational powers of e, *Acta Arith.* **27**, (1975), 61–87 **[Math. Rev. 51#3070], [RNT I25-211]**

[194] A necessary and sufficient condition for transcendency, *Math. Comp.* **29**, (1975), 145–153 **[Math. Rev. 52#3072], [RNT I25-214]**

[195] On the transcendency of the solutions of a special class of functional equations, *Bull. Austral. Math. Soc.* **13**, (1975), 389–410 **[Math. Rev. 53#2850], [RNT I65-205]**

[196] On certain non-archimedean functions analogous to complex analytic functions, *Bull. Austral. Math. Soc.* **14**, (1976), 23–36 **[Math. Rev. 53#8025], [RNT Q25-225]**

[197] An addition to a note of mine, *Bull. Austral. Math. Soc.* **14**, (1976), 397–398 **[Math. Rev. 54#2596], [RNT H25-202E]**

[198] A theorem on Diophantine approximations, *Bull. Austral. Math. Soc.* **14**, (1976), 463–465 **[Math. Rev. 54#12667], [RNT J04-209]**

[199] Corrigendum to "On the transcendency of the solutions of a special class of functional equations", *Bull. Austral. Math. Soc.* **15**, (1976), 477–478 **[Math. Rev. 54#12674], [RNT I65-205E]**

[200] *Lectures on Transcendental Numbers*, B. Divis and W. J. LeVeque, eds., Lecture Notes in Mathematics **546**, Springer-Verlag, Berlin/New York, (1976), **[Math. Rev. 58#1077], [RNT I02-200E]**

[201] On a class of non-linear functional equations connected with modular functions, *J. Austral. Math. Soc. Ser. A* **22**, (1976), 65–118 **[Math. Rev. 56#258], [RNT F10-229]**

[202] On a class of transcendental decimal fractions, *Comm. Pure Appl. Math.* **29**, (1976), 717–725 **[Math. Rev. 54#12675], [RNT I30-214]**

[203] On some special decimal fractions, in Hans Zassenhaus ed., *Number Theory and Algebra*, Academic Press, New York, 1977, (1977), 209–214 **[Math. Rev. 58#539], [RNT I25-230]**

[204] On a special function, *J. Number Theory* **12**, (1980), 20–26 **[Math. Rev. 81c:30006]**

[205] *p-Adic Numbers and Their Functions* (2nd edition of "Introduction to p-adic Numbers and Their Functions"), Cambridge Tracts in Mathematics **76**, Cambridge University Press, London/New York, (1981), **[Math. Rev. 83i:12002], [RNT S02-201E]**

[206] On two definitions of the integral of a p-adic function, *Acta Arith.* **47**, (1980), 105–109 **[Math. Rev. 82b:26024], [RNT Q25-281]**

[207] On some irrational decimal fractions, *J. Number Theory* **13**, (1981), 268–269 **[Math. Rev. 82j:10061], [RNT I15-233]**

[208] On a special non-linear functional equation, *Philos. Trans. Roy. Soc. London Ser. A* **378**, (1981), 155–178 **[Math. Rev. 82m:10042], [RNT F10-229E]**

[209] Fifty years as a mathematician, *J. Number Theory* **14**, (1982), 121–155 **[Math. Rev. 84a:01051]** Remark: The present publication list is a correction and extension of the one appearing here.

[210] On a special transcendental number, *Arithmétix* **5**, (1982), 18–32 Remark: Information bulletin of number theorists in France

[211] On the zeros of a special sequence of polynomials, *Math. Comp.* **39**, (1982), 207–212 **[Math. Rev. 83k:12019]**

[212] On a theorem in the geometry of numbers in a space of Laurent series, *J. Number Theory* **17**, (1983), 403–416 **[Math. Rev. 85e:11043]**

[213] On the analytic solution of certain functional and difference equations, *Proc. Roy. Soc. London Ser. A* **389**, (1983), 1–13 **[Math. Rev. 85e:30041]**

[214] Warum ich eine besondere Vorliebe für die Mathematik habe, *Jbericht d. Deutschen Math.-Verein.* **85**, (1983), 50–53 **[Math. Rev. 84f:01073]** Remark: Essay originally written in 1923

[215] On Thue's theorem, *Math. Scand.* **55**, (1984), 188–200 **[Math. Rev. 86h:11030]**

[216] Some suggestions for further research, *Bull. Austral. Math. Soc.* **29**, (1984), 101-108 **[Math. Rev. 85e:11043]**

[217] (with D. H. Lehmer and A. J. van der Poorten) Integers with digits 0 or 1, *Math. Comp.* **46**, (1986), 683–689 **[Math. Rev. 87e:11017]**

[218] A new transfer principle in the geometry of numbers, *J. Number Theory* **24**, (1986), 20–34 **[Math. Rev. 88e:11054]**

[219] The successive minima in the geometry of numbers and the distinction between algebraic and transcendental numbers, *J. Number Theory* **22**, (1986), 147–160 **[Math. Rev. 87d:11043]**
[220] On two analytic functions, *Acta. Arith.* **XLIX**, (1987), 15–20 **[Math. Rev. 89f:30006]**
[221] The representation of squares to the base 3, *Acta Arith.* **LIII**, (1989), 99–106

Appendix II

Fifty Years as a Mathematician II
Kurt Mahler

Abstract

Kurt Mahler FRS, FAA died on 26 February, 1988 after a long and distinguished career devoted primarily to the theory of numbers. The present material is copied from a typescript entitled 'Fifty Years as a Mathematician' written principally in 1971. Remarks [appearing as insertions in the text] are his subsequent handwritten additions. I have made no changes to the manuscript {other than for a few notes} and the minor change in title; in particular, I have adhered to its punctuation and capitalisations. The interested reader will also want to consult Mahler's biographical articles: 'Fifty Years as a Mathematician' (*J. Number Theory* **14** (1982), 121–155)—a rather more mathematical version of the present notes; 'How I became a mathematician' (*Amer. Math. Monthly* **81** (1974), 981–83); and an essay written in the twenties: 'Warum ich eine besondere Vorliebe für die Mathematik habe' (*Jber. d. Deutschen Math.-Verein.* **85** (1983), 50–53). *A. J. van der Poorten* (Macquarie University)

As I have reached the age of 68, it is perhaps appropriate to look back at earlier times in my life and to my occupation with mathematics during these many years.

My family had no academic traditions or connections. All my four grandparents came from small places in the Rhineland in Western Germany, and none of them went to more than an elementary school (*Volksschule*). As Jews they had additional difficulties in the old Prussia of the nineteenth century.

My father and several of my uncles went into the printing and bookbinding trade, beginning at the bottom as apprentices and saving slowly enough money to start small firms of their own early this century.

My parents had eight children, but as was usual in those days four of them died young. My twin sister and I were born in 1903 and were the youngest children. There was also an elder sister who at present is still alive and with her husband and one daughter lives at Venlo in the Netherlands [Lydia Krohne died at the age of 89 in 1984; her husband died about 10 years earlier, but her daughter is still alive in 1986]; and there was an elder brother who with his wife disappeared in a Nazi concentration camp during the second world war.

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Like my father, we four surviving children were avid book readers, in a somewhat omnivorous sense. My brother became a printer like his father {Josef joined and eventually took over his father's firm.}, and my elder sister, who was very musical and wanted to become a singer, ended by marrying a printer who was also a musician. My twin sister was perhaps the most capable of us four, at least in business matters. She was excellent at languages, spoke and wrote French and Spanish, and repeatedly visited South America. She finally married into a dry-cleaning firm in Thuringia in Central Germany and made it prosper, but she died in 1934 {Elsewhere, Mahler describes Hilde as being driven to suicide by Nazi persecution.} and her only daughter in 1963.

My sisters and my brother were physically healthy and strong. I was not, but from early childhood suffered from a surgical form of tuberculosis in the right knee. In those days there was no effective treatment of this disease, and it was not healed until I was 35 when the knee cap, after several earlier operations, was removed successfully. Due to my illness I went only for a total of four years to school; for an additional two years I had to stay at home and received some elementary lessons in the three R's. At this time the opinion of my private teacher was that I was "not stupid, but lazy".

I left the elementary school shortly before I was 14 and for the next two years attended day courses at two elementary technical schools as my parents thought that I should try to become a fine-mechanic {This is an over direct translation of 'Fein Mechaniker' which Mahler translates elsewhere as 'precision mechanic'. The point was that this was a job that could be done sitting down.}. However, these technical schools gave me the first instruction in elementary algebra and geometry, and I immediately decided that this is what I really liked. So, during the summer vacation of 1917 when I had just become 14, I bought a logarithm table and enjoyed myself doing problems, and shortly afterwards taught myself some plane and spherical trigonometry. I next acquired a good book on analytic geometry and, in 1918, began with differential and integral calculus. About this time I entered a machine factory in my home town Krefeld as an apprentice, working first for one year at their drawing office and then for not quite two years in the factory itself. The reason for this change of plans was that an apprenticeship of this kind perhaps might enable me to go to a technical university (*Technische Hochschule*) for the study of mathematics. Such a study would not require the very difficult University Entrance Examination (*Arbiturienten-Examen*). Actually, as I shall explain, I did in the end pass this examination!

When returning home from the factory, during my period at the machine factory, I attended evening classes on technical courses, and these included some more elementary mathematics. However, most of my free time I spent on my mathematical readings. I bought a good course on advanced calculus (Césaro-Kowalewski, *Infinitesimalrechnung*) and the first volume of

Pascal's *Repertorium der Mathematik.* From the first work I learned roughly the contents of a first year honours algebra and calculus course, and from the second one, which gave only results and no proofs, and was very hard reading, I learned about groups, matrices, invariants, and other topics. The next books I acquired were some volumes of Bachmann's *Zahlentheorie*, Landau's *Primzahlen*, Knopp's little *Funktionentheorie*, and the Klein-Fricke volumes on the *Ikosaeder*, *Modulfunktionen*, and *Automorphe Funktionen*; further *Hilbert's Grundlagen der Geometrie* and advanced texts on Analytic geometry. Thus I became acquainted with the different types of non-euclidean geometry. Naturally, at this time I had none to help me with those studies. But I met fellow students at the evening classes, and I tried to teach one or two of them some of the pretty things which I had just learned and hoped to understand better by talking about them!

The great moment came about 1921. My father, without my knowledge, sent some of my mathematical attempts to the director of the Realschule in Krefeld Dr. J. Junker who was then about 50. He had originally been a student of Christoffel and had obtained his Ph.D. in the 1890's with a thesis in invariant theory. Dr. Junker immediately decided to help me. He repeatedly sent my attempts in mathematics to Felix Klein in Göttingen who gave them to his assistants for a report. C. L. Siegel, who at about this time began his great career in Göttingen gave the first report and suggested that I should be helped to pass the university entrance examination and so be enabled to go to the university.

So I left the factory and for the next two years took lessons in certain required subjects, in particular German, French, and English. These lessons were given by teachers at Dr. Junker's school and naturally were a great help. They did not stop my private studies in mathematics which by now dealt with subjects at the university level.

The Prussian government which was very liberal at this time allowed me to take the examination in my home town Krefeld. I did so in the fall of 1923, during one of the worst periods of the big German Inflation and while there was a lot of political trouble due to the Ruhr occupation. (Needless to say that I was at this time and long into the 1930's still a very patriotic German!) There were just two of us external students who took the examination. It lasted from Monday Morning to Friday Afternoon and covered in written papers the three languages, mathematics, and physics; and on Friday afternoon there was an additional oral in these and some other subjects. My fellow sufferer was then told that he had failed, but I just scraped through!

Thus, when I had reached the age of 20, I was able to go to the university. I had originally intended to study in Göttingen. But at about this time Siegel had become professor at the university of Frankfurt, and he invited me to

https://doi.org/10.1017/S1446788700034558 Published online by Cambridge University Press

come there for my first semesters. Frankfurt had the advantage of having only few students, while the mathematical faculty consisted at this time of Dehn, Hellinger, Epstein, Szass, and Siegel, all distinguished in their subjects. Also the department at this time was the only German university to hold a weekly seminar on the history of mathematics.

I well remember my first semester. I took the second part of calculus given by Siegel (I never took the first part, but gave it myself later many times at Manchester); a very difficult course on *analysis situs* (topology) by Dehn; elliptic functions by Hellinger; and some non-mathematical lectures which I liked much less. In addition I took part in the historical seminar [History of Mathematics], and also in a seminar on *Kreisteilung* (cyclotomics) where I in fact gave several of the lectures. It was at this time the rule that every member of the Seminar had to write a short paper for the Seminar Book of the Frankfurt Mathematical Department, and so several of my contributions can be found in it. [This book unfortunately was lost during the Second World War. Copies are with the Hebrew University in Jerusalem.]

After Easter 1925 Prof. Siegel went on leave for foreign travel and studies. During the three semesters till then he had given me much personal help and supervision and taught me methods useful for my research. I therefore went in the summer of 1925 to the University of Göttingen and stayed there in fact until the Nazi revolution of 1933.

Göttingen was then still the centre of world mathematics. Klein died shortly after my arrival. But Hilbert was still working and lecturing, and there were Landau, Courant, Emmy Noether, Ostrowski, and at a slightly later date Herglotz and Weyl. I had little contact with Landau at this time since he was then giving a calculus course for beginners. At a later date I belonged to the group of advanced students who helped with his students, and he became more interested in my work. Our personal relations were always friendly, and I enjoyed parties at his house.

Herglotz gave the most perfect lectures, usually on subjects on the boundary between pure and applied mathematics. It was difficult to add anything to what he said about his problems; so he had few research students.

Quite different were the lectures by Courant. I found them rather vague and was never quite sure whether he had in fact proved his assertions. However, I learned from him about the direct methods in the calculus of variations, and many years later this enabled me to apply analogous ideas in the geometry of numbers of general point sets.

I learned also much from Emmy Noether. Her lectures were usually badly prepared, and she spoke much too fast and wiped the blackboard far too often. Most of the time I could not understand her during her lecture, and I had to allow her ideas to simmer in my head for quite a period before

I could understand them. But I learned from her about general fields and in particular the p-adic field, and was in later years to apply these notions many times. I never took to the axiomatic method; it seemed to be so much more fun to deal with naturally occurring rather than constructed problems in mathematics.

During my stay in Göttingen, there were many foreign visitors, topologists like Alexandroff and Hopf, algebraists like van der Waerden and A. Weil, number theoreticians like Schnirelmann and Gelfond, and many others. Let me mention in particular Norbert Wiener because I acted as his (unpaid) assistant in 1926 when his German was not yet as good as in later years {A result of this assistantship was Mahler's first paper [1] which appears, in English! as an appendix to a paper of Wiener}; L. J. Mordell who was to play such an important role in my life; and in the early 30's my dear friends J. F. Koksma and J. Popken from the Netherlands.

I became very ill in 1927 and had one kidney removed. This kept me at home in Krefeld for the summer semester of 1927 and continued to trouble me for several more years. The illness delayed my Ph.D. examination for several months, until it took place in December 1927 at the University of Frankfurt. The thesis {'Ueber die Nullstellen der unvollständigen Gammafunktionen' with Referent: Prof. Dr. O. Száss and Korreferent: Prof. Dr. C. Siegel; dedication: Herrn Professor Dr. Josef Junker in Dankbarheit und Verehrung gewidmet} dealt with the zeros of the incomplete Gamma Functions, thus with the zeros of a class of functions tending to a function without zeros. Ostrowski was not very impressed with this work and told me to do less easy mathematics. I hope that I have followed his advice at least in some of my later research.

While I was ill at home in 1927, I succeeded in proving the transcendency of $z + z^2 + z^4 + z^8 + \cdots$ for algebraic z satisfying $0 < |z| < 1$. The method was new and depended on the functional equation $f(z) = z + f(z^2)$ for the series. Several papers of mine generalised this result, and I could in particular establish the transcendency of $\sum_{n=1}^{\infty}[n\alpha]z^n$ for all real quadratic irrationalities α and all algebraic numbers z satisfying $0 < |z| < 1$; in fact, all derivatives of this function of z have the same transcendency property.

E. Landau did not show much interest in these results. So I next turned to a closer study of the approximation properties of e and π. I found my classification of the transcendental numbers into the three classes S, T, and U. It allowed me to prove that none of the numbers e, π, or $\log 2$ can be algebraically dependent on a Liouville number. This settled a question which Perron had asked in his book '*Irrationalzahlen*'. It was also this work which brought me into close contact with Koksma and Popken when they spent a summer semester in Göttingen in the late {This must be a mistype for 'early'.} 1930's.

While still in Frankfurt, I learned from Siegel his proof and improvement of Thue's theorem on the approximation of algebraic numbers, and I had studied his earlier papers on this subject and also his great Academy Paper on lattice points on algebraic curves. I naturally was also acquainted with the treatment of Thue's theorem in Landau's *Vorlesungen*. I think it was during the Whitsun holidays of 1930 when bad weather forced me to stay at my lodgings on a small North Sea island that I got the idea of establishing an analogous result for p-adic numbers, for I had at last learned how to handle them. I could easily extend my classification of S, T, and U-numbers to p-adic numbers, but at that time was not yet able to decide whether the p-adic exponential function e^z, assumed to be convergent, was transcendental for algebraic $z \neq 0$.

In the autumn of that year I attended for the first time a meeting of the Deutsche Mathematiker-Vereiningung and there reported on these p-adic problems and what I had so far been able to prove. Not long afterwards I learned how to generalise my p-adic analogue of the Thue-Siegel theorem to the case when roots of the same algebraic equation in several of the p-adic completions and in the real completion of the rational field are approximated simultaneously by the same rational number. This leads to results like the following one. If $F(x, y)$ is an irreducible binary form of degree at least 3 with integral coefficients, then the number of integral solutions x, y of $F(x, y) = k$, $(x, y) = 1$, does not exceed c^{t+1}, where $c > 0$ does not depend on k, and t denotes the number of distinct prime factors of k. Siegel called these results "schön und wertvoll", and I myself considered them as my first important contribution to mathematics.

I lectured on these p-adic results before K. Hensel in Marburg who was very pleased to learn of a new application of his p-adic numbers; and I gave similar lectures in the seminar of I. Schur at Berlin. During this visit to Berlin, von Papen came to power, so preparing for Hitler.

It was during these last years at Göttingen, shortly before I reached the age of 30, that I had my first income (until then, my parents and even more members of the Krefeld Jewish Community had helped me with my studies). I was given a research fellowship by the Notgemeinschaft der Deutschen Wissenschaften of not quite \$500 for a little more than two years. This was for me a very large income, and I could save some of it for the coming troublesome years. Also, about the end of 1932, I was appointed to an assistantship in the Mathematics Department of the University of Königsberg in the far East of Germany. It was to start in the summer of 1933, but naturally I never took it up.

Hitler came to power in early 1933, and I knew at once that I had to find a home abroad. During the summer of 1933 I went for six weeks to Amsterdam for research and for discussions with Koksma and Popken and with van der Corput who had been their teacher. About this time Mordell obtained for me a small research fellowship and invited me to come to Manchester during the session 1933–34 which I did in September of that year.

During this first visit to Great Britain my English was still somewhat defective. However, immediately on my arrival I was put before a blackboard and told to give a seminar lecture, a drastic but helpful method! I still have a suspicion that my listeners suffered even more from the lecture than I did! Until then, I only had a reading knowledge of English. Now I slowly learned to speak and understand it, and even to understand the language as spoken by the Mancunians.

During this stay at Manchester I learned that, thanks to van der Corput I had been given a stipend by a Dutch Jewish group to enable me to work for the next two years at the University of Groningen in the East of the Netherlands. I went to Groningen first for a short visit in May to June 1934 and then stayed there from the fall of 1934 till the summer of 1936. I worked in the Department of Mathematics of Groningen University. I did mainly research, but I also gave during the two years a course on recent work in Diophantine Approximations and Minkowski's Geometry of Numbers. Regrettably, the rules of the university did not allow me to attend the Dutch lectures in the department without inscribing as a student; therefore I never learned to speak Dutch well, but only acquired a reading knowledge.

My research at Groningen dealt with pseudo-valuations; with the p-adic analogue of the Gelfond-Schneider theorem on the transcendency of α^{β}; with geometry of numbers and its applications to transfer theorems; and with the Taylor coefficients of rational functions.

Towards the end of my stay at Groningen I was run into by a bicycle rider. This was disastrous, for the tuberculosis in my right knee became again active. I was unable to walk for some time and had to stay at Krefeld. There I had several operations, and finally the right knee cap was removed. This had the desired effect of removing the infection; it died out completely in the next few years.

The period after the last operation was very painful, and for several months early in 1937 I was given injections of morphine. At the end of this time when the pain had lessened and the injections were stopped, I found that the drug had not affected my brain, for I could show the transcendency of the decimal fraction $0.1234\ldots$ {That is $0.123456789101112131 4\ldots$ } and of infinitely many similar fractions. The proof depended on a generalisation of a theorem by Schneider on the rational approximations of algebraic numbers.

During the summers of 1937 and 1938 I spent three months each in a hospital for sun treatment of tuberculosis at Montana, Valais, Switzerland. This treatment probably contributed much to the final recovery from my disease.

Afterwards, in the fall of 1937, I returned to Manchester University at the invitation of Mordell to spend the remainder of my small fellowship. As events were to show, I remained at this university until 1963, a total of 26 years.

I depended originally on the small Bishop Harvey Goodwin Fellowship. However, shortly after my return in 1937, a post became vacant in the department and I was appointed for the session as a temporary assistant lecturer. For this the British Government had to give its permission because at this time there still severe restrictions on the employment of aliens.

Thus, at the age of 34, I started on my first lecture courses to British Undergraduates. My English at this time had slightly improved since 1933, but still left much to be desired. Fortunately there were few people round me who spoke German, and so I had perforce to speak, write, and learn to understand English and to acquire its rather difficult idiom.

The period from the fall of 1937 to the summer of 1940 and the end of the phony war was of great importance for the progress of number theory at Manchester University. Mordell's seminar on number theory included at this time for variable periods mathematicians like Davenport, Young, Erdős, Chao Ko, Lehmer, Billing, Zelinskas, and myself. All the new progress in number theory was discussed at the weekly seminar, and much was contributed to this progress by its members. The outstanding event was a series of papers by Davenport on the products of three linear forms in three integral variables; he established the exact minima when either all three forms were real, or when one form was real and the other two were complex conjugate. This was the first breakthrough in a more-dimensional non-convex problem in the geometry of numbers. Davenport himself of course used arithmetical rather than geometrical methods. But in 1940 Mordell found a geometrical approach to these questions and so could give a new proof for Davenport's results. In the next years Mordell could generalise these ideas and apply them to general classes of two-dimensional non-convex star domains. Davenport, who in the fall of 1941 had gone to Bangor, also made further important contributions to their theory.

Let me go back to 1938. When I had concluded my university session as a temporary assistant lecturer, I worked for a term again on research alone. But then, for the last two terms of the session of 1938-39, I once more had an appointment at Manchester as a temporary assistant lecturer. But between the fall of 1939 and that of 1941 I had no appointment and did research, while living on the balance of my small fellowship and on my savings from the two lecturerships.

At the summer of 1939 Chao Ko had just gone back to China where he was a professor at the University of Szechuan, and through his efforts I was offered a professorship at this university. China was of course at this time at war with Japan. I accepted the appointment; but my internment in the following summer and the war difficulties made the journey to Western China impossible. However, I became interested in Chinese and for the session 1938-39 had one hour a week instruction by the Reader in Chinese at Manchester University. I never learned to speak. But I acquired enough characters, grammar, and idiom, in the following years to be able to read a little. The reading matter consisted mainly of the old novels like the '*History of the three Kingdoms*', and of some history. In this way I obtained perhaps a slightly better understanding of the Chinese.

From June to September 1939, into the outbreak of war, I had a glorious vacation on a small island in the Scilly Isles west of Southern England. It was good for my health, and I also could do some research. In particular, I could at last solve a small problem in combinatorial analysis which had puzzled me since my Frankfurt days. Other research at this time and for the next few years dealt with special classes of Diophantine equations, with Hlawka's theorem on the product of two linear polynomials in two variables with complex coefficients, and with the related problem for quaternions, etc.

As I said already, I was unemployed during the first two years of the war and spent my time on research. Like most other German refugees I was interned for three months during the summer of 1940, a rather uncomfortable time. During the first six weeks, when it was very rainy, we were in a camp under tents at the border of Wales; but later we lived in empty boarding houses on the Isle of Man. Here a group of us former university lecturers and students got together and we opened a university for the interned! I myself gave a course on the construction of real numbers by means of Cauchy sequences of rational numbers. Later on, I started many first year honours analysis courses at Manchester University in this fashion.

After my return from internment in the fall of 1940 I began to work on Farey Sections in complex number fields. This work was completed only after the war with the cooperation by Ledermann and Cassels and published in a joint paper. At about this time I received an invitation as lecturer to the University of Cape Town where L.C. Young was then professor of mathematics. On his advice I declined this invitation.

For I knew then already that Davenport, who until 1941 was an assistant lecturer at Manchester, but who had been elected to the Royal Society before the war, was to leave for his professorship at Bangor. The British Government allowed my appointment to the vacant post as assistant lecturer, and

I remained on the staff of Manchester University from the fall of 1941 until that of 1963, successively as assistant lecturer, lecturer, senior lecturer, reader, and finally as personal professor. The professorship was the first such appointment to be made in the mathematics department of Manchester University.

I think it was about 1941 when, as a consequence of the work by Davenport and Mordell already mentioned, I myself became interested in the general geometry of numbers of non-convex sets. My first, rather long, paper on the subject dealt with the case of bounded two-dimensional star domains, and for these I gave a finite algorithm for finding their critical lattices and the lattice constant. I finished this paper while I was on Xmas vacation at a boarding house in a small village near Lancaster, and while there divided my time most agreeably between working, walking, and reading. On later visits I was never again as successful in combining these occupations!

I soon extended my results to the case of unbounded star domains, and this led logically to the study of lattice point problems for any n-dimensional star bodies and even more general sets. My compactness theorem for bounded sets of lattices was the main tool in these investigations and led to existence and other general results. This was perhaps the second time that I made an important contribution to mathematics. My method was soon adopted by other mathematicians, in particular by Davenport, C.A. Rogers, and I.W.S. Cassels, who all used it with great success.

When the war ended in 1945, I applied for British Naturalisation. The application was granted in 1946, and in 1948 I was elected to the Royal Society. At about this time, Mordell had already left for Cambridge, and M. H. A. Newman had become his successor as head of the Mathematics Department at Manchester.

The next year, from January to September 1949, I had my first leave from Manchester University. I made then my first visit to the United States. For the greater part of my leave I went to the Princeton Institute of Advanced Study. But during the summer, when the heat and humidity of Princeton became intolerable, I went for a few days to the University of Colorado, for a week to the University of California at Los Angeles, for six weeks to the University of California at Berkeley, and for three weeks to the University of British Columbia at Vancouver. At Boulder, Colorado, B. W. Jones invited me to come again the next year during the summer vacation and to give then a course on the geometry of numbers, before attending the International Congress of Mathematicians due to take place at Harvard in the fall of 1950.

I almost did not make it. For during the Xmas vacation of 1949–50 I contracted Diphtheria, and I returned to Manchester from the Netherlands suffering from this disease and all kinds of complications, including pneumonia. As a child I had of course never been immunised against diphtheria because this was not yet done at the beginning of the century. I was ill for some three months, but thanks to antitoxin, sulfonamides, and penicillin, I finally recovered. However, immediately afterwards I was attacked by a delayed consequence of the diphtheria, and I had to endure another three months suffering of peripheral neuritis which was almost worse and certainly more painful. Fortunately, I recovered from it just in time to make my proposed trip to the University of Colorado. I spent a most delightful summer there lecturing on geometry of numbers and making many trips into the Rocky mountains. Afterwards I went to the International Mathematical Congress at Harvard. This was the third international congress which I attended. For before the war I had already gone to the congresses at Zurich and at Oslo; in later years I was also to attend the congresses at Amsterdam and Edinburgh.

During the next seven years I stayed at Manchester without any long leaves. But I made a long visit to the Hebrew University in 1951, to the University of Göttingen in 1952, to that of Brussels in 1953, and to that of Vienna in 1954, always during the vacations. During the long vacations in these years I usually went for up to 7 weeks to some seaside place in Great Britain, later in particular to Herm Island in the Channel Isles. In those days this island was still very quiet and unspoiled, but as I found out this year on a short visit, it is no longer so.

My research during these years dealt with various questions. I mention a generalisation of Siegel's theorem on lattice points on algebraic curves, and some general results on the geometry of numbers of compound and associated convex bodies. The latter have recently been applied to simultaneous rational approximations of algebraic numbers by W. Schmidt.

Then, during the spring and fall of 1957, I was once again granted a long leave by Manchester University. For the first two months I had an appointment as traveling lecturer for the Mathematical Association of America, visiting a great number of colleges and universities in the Eastern half of the U.S.A. The summer I spent once again at the University of Colorado, and during the fall I was for the first time at the University of Notre Dame. I lectured at both places on the p-adic generalisations of Roth's theorem on the rational approximations of algebraic numbers. By early 1959 I had collected these lectures into a little book which finally appeared after a long delay in 1961 at the University of Notre Dame Press (*Diophantine Approximation* I; the second part has not so far appeared and probably never will!).

At Notre Dame I found a friend in A. E. Ross who was then the head of the Mathematics Department. On his invitation I went during several summers in the next years as a visiting professor to Notre Dame, until 1963 when I left Manchester for Australia, and Ross soon after went as head of the Department of mathematics to the Ohio State University at Columbus.

Since 1938 I had, except for a short interruption in 1940–41, been living at Donner House, Fallowfield, Manchester. This was a very comfortable house for Staff, Research workers, and Visitors, of the University of Manchester. It was lying among trees and not far from a pretty park, and it had the great advantage of bringing together members of different departments of the university. But towards the end of the 1950's the University decided to tear down Donner House and use the grounds for Student Halls (which turned out to be rather ugly and uncomfortable!). So, in 1958, I bought a small semi-detached house in Withington and lived there by myself until 1963.

At about this time most of my colleagues in the Mathematics Department started to accept appointments at other universities, and it became for me rather lonely in Manchester; in particular, there was now nobody left with interest in number theory.

Once again I was given leave by the University of Manchester from the summer of 1962 to early 1963. This time I spent only short periods at the Universities of Colorado and Notre Dame, but for the first time went from October 1962 until the end of the year to the Institute of Advanced Studies at the Australian National University in Canberra, Australia. Here my former colleague B. H. Neumann had become the head of the Mathematics Department at the I.A.S.. I immediately took to Australia and to Canberra and was very happy to accept a professorship at the Institute to start in September 1963. The sunny climate and the beautiful situation were so different from the ugliness of Manchester. At this time, Canberra did not yet have its lake; it was filled only after my arrival, early in 1964.

I took up my new appointment in September 1963 and I remained at the Australian National University for five years, until I reached the rather early retirement age limit of 65 in 1968. It was a very stimulating and useful time, and I liked very much my Australian colleagues at the different Australian universities. When I was not on short vacations or at meetings of the Australian Mathematical Society, I was working on research at the Institute in Canberra.

There was at this time no teaching of number theory in the undergraduate school (School of General Studies) at the ANU. I therefore gave a course on this subject to second and third year students at the SGS, probably the first one ever in Canberra. One of my undergraduates, Coates, asked me to introduce him to research. I provided him with problems to work on, and by the time he obtained his B.Sc., he had already several papers published or in print. He is now a Ph.D. and has a professorship at Harvard, and he has published a number of very good papers dealing with Diophantine Approximations and Transcendency. [Coates was elected to the Royal Society in 1985 and became the successor of Cassels at Cambridge.]

My own work at Canberra concerned mainly two subjects. One was the global approximation theory of algebraic numbers in a number field. My paper J. Austral. Math. Soc. 4 (1964), 425–448 should be useful for the deeper study of such fields; but the super-abstract algebraists of to-day probably find it too specialised! [It has since then found applications.]

My other work centered on transcendental numbers. The fundamental papers by Shidlovski, and by Baker, became available at this time. I carefully worked through them and somewhat simplified their methods. I prepared detailed lecture notes which, perhaps, some day will become a book. { They did, see [200].}

One fruit of my study of Shidlovski's papers was that, during a short week in 1968, I thought I had proved the transcendency of Euler's Constant. But the proof was false, and I could only prove a much less interesting result!

During the American summer in 1965 I once again was at the University of Colorado, and I also made a short visit to Ohio State University where Ross was now the Head of the Mathematics Department. Two years later I spent four months, from September 1967 to January 1968, at the University of Arizona at Tucson, lecturing on transcendental numbers, and afterwards, for about two months, visited again Ohio State University. It was at this time that I accepted an appointment as professor at OSU, to start in the fall of 1968 when my appointment at Canberra was due to end. Since the fall of 1968 I have in fact been here in Columbus at OSU. But I was on leave for three months, from November 1968 to March 1968, and again this year (1971) from the end of March to that of June. The first leave I spent again at the ANU in Canberra, and the second one I used to lecture at the University of Sussex, Brighton, in England.

At this moment, I expect to spend the time from November 1971 to March 1972 at the University of California at San Diego (La Jolla), and then to return to Canberra for my final retirement. I am feeling my age more and more and am troubled by my eyes; so it is perhaps time for it.

While still in Canberra, I was in 1965 elected to the Australian Academy of Science. And in June of this year I was awarded the de Morgan Medal of the London Mathematical Society.

It was a great loss to me when shortly one after the other Koksma, Davenport and Popken died in recent years. All three were dear friends to me for many years.

As a mathematician, my greatest debt was due to Siegel and Mordell. The first taught me much in research, and the second one in teaching and the writing of mathematics. Until Mordell left Manchester in 1945, he corrected the English and the mathematical style of almost all my papers. But in exchange I used to prepare the drawings for his papers in geometry of numbers!

During my fifty years as a mathematician, I have seen immense progress in mathematics and in all the more exact parts of science, including the biological sciences. I feel that in almost everything else there has been a regression. With the extremists on both sides of the spectrum trying to destroy basic research and replacing it by their dogmas of intolerance and power madness, we seem to be on the way to new dark ages.

COLUMBUS, 9 SEPTEMBER, 1971

Addenda

While in the U.S.A. last year (1973), I bought an HP45 electronic calculator and have become interested in computers. Our department at IAS, ANU is due to obtain for me the improved programmable HP65 calculator, and I am looking forward to working with it. It is amazing how much progress in this domain has been made and how prices are coming down, this at a time of big inflation.

I was very pleased by all this progress in Astronomy and Astrophysics during the last years. It gave me particular pleasure to learn that Mars once seems to have had a lot of water and may perhaps become again livable. I was also glad that the schoolbook teachings about the noble gases were disproved when the first compounds of these were obtained.

The progress in my lifetime in the exact sciences, including biology has been impressive, and the years of this century must be unique in history in this regard. Unfortunately, no such progress has been made in the social sciences and certainly not in economics and politics in general. Our world would be in a better state if all the politicians of all the countries could be collected and sent to the moon! And the same should be done to all those representatives of the news media and all the intolerant dogmatists of religion. One needs only think of the two world wars and of the present events in Ireland to see how little christianity does for morality; and the events in Pakistan, not to mention the Near East, prove the same for Islam. Our world would be in a better state if the teachings of Confucius were applied!

APRIL 1974

In 1976 I bought a Texas Instruments RE 52 Calculator, and in 1977 a TI 59. These programmable calculators are very powerful and have been used by me to study the solutions of $h(z^2) - h(z)^2 + c = 0$ which have a simple pole of residue 1 at $z = 0$. I have not tried to learn to work at computers.

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