

with no such lapidary statements concerning the exclusive merit of one or the other, as are to be found even in quite recent books on History of Mathematics (cf. J. E. Hofmann, *Geschichte der Mathematik II*, pp. 62, 70; Berlin 1957, Sammlung Göschen 875; reviewer's quotation). The more recent developments in Analysis are covered in the subsequent essays, mainly "Espaces vectoriels topologiques" and "Intégration".

For every essential statement a reference is given to the literature, collected in the bibliography at the end of the book; 253 authors, many being represented by several items. Most historical references are of course to the standard works by Neugebauer, Tropicke, etc., and to the collected works of the great mathematicians of the past. But also outstanding papers of a great number of more recent and contemporary authors are referred to and in some cases discussed.

In conclusion: this is a book on the history of mathematics as the active as well as the teaching mathematician wants it; to the best knowledge of the reviewer, nothing similar has been available so far. If anything remains to be desired it is, that "the greatest mathematician of our time" should soon find it possible to complete his work by a second volume dealing with those topics that had to be omitted in the present volume.

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Calculus, by R. L. Jeffery. Third edition, University of Toronto Press, 1960. xii + 298 pp. \$4.95.

There are many students who must learn calculus, who are indeed capable of using it effectively in their several sciences, but for whom a full treatment of ϵ 's and δ 's at the very beginning is too hard. In the middle of the twentieth century these students need something considerably less unrigorous than books like the famous "Calculus made easy": the present book bridges the gap. Although the initial stages are largely intuitive, rigour being postponed until just before the treatment of series (which is pointless if unrigorous), theorems are correctly stated: e. g. we read that a continuous function attains its bounds on a closed interval, the words "continuous" and "closed" not being omitted. Similarly the existence of the definite integral is stated (and later proved) for continuous functions: the student is carefully warned that he cannot rely on every function being integrable. After a preliminary review of the number-system and the concept of function, differentiation is motivated by a discussion of speed, leading to a very brief discussion of limits. Next comes some practical differentiation and then differentials and anti-differentials. Here, conveniently earlier than usual, we meet separable differential equations. The anti-differential is the primitive in differential (rather than derivative) notation, and the concept is very properly divorced from that of integral. (Would that the author had

equally clearly divorced differentials from increments.) Next comes the definite integral, motivated by area, still treated intuitively but very clearly not made to depend on geometry; and then several very standard chapters: transcendental functions; the usual applications of differentiation to problems, maxima and minima, and graphs; applications to mechanics; and some practical integration. The first part of the book finishes with the mean-value theorem and various topics, principally rectification and L'Hopital's rule, whose common ground is that they depend thereon.

Now come the fundamentals of the subject, based on the theorem that every set which has an upper bound has a least one, which is proved using infinite decimals. This leads to a rather brief chapter on series. The author now turns back to the practical aspect, with functions of several variables, including multiple integration. Because this is very non-rigorous (there is no explanation, let alone a definition, of what is meant by " (x, y) tends to (x_0, y_0) ") it might well have come before the fundamentals. This treatment of partial derivatives is the shortest which the reviewer has ever seen, and because experience shows that pupils find this topic difficult, he feels that it might be a weak point of the book. The book ends with four chapters on vectors (two of which are pure linear algebra) and one on quadrics.

The writing is lucid with felicitous touches. Few writers can have explained that integration is harder than differentiation because it is not a procedure but the inverse of a procedure, in as succinct a phrase as "[Antidifferentiation is] turning the wheels of differentiation backwards, and wheels do not always turn backwards easily".

The reviewer liked the treatment of physical quantities. The author does not define moment, for example, by an integral. He postulates that it is additive and monotonic and then shows that (for continuous distributions) it must be given by the integral. The reviewer also likes the use of $D_x y$ to denote the derivative of y with respect to x , leaving dy/dx to denote the quotient of dy by dx . He did not, however, like the definition of differential, which does not allow dx to be zero without dy being zero, and so leads to grave trouble at points where the tangent to a curve is parallel to the y -axis. Indeed, on this definition differentials have no advantage over derivatives for functions of one variable.

The part of the book which the reviewer liked least is the treatment of maxima and minima, which contains a mechanical application of first- and second-derivative tests, although the examples (and problems) can be solved by using the sign of the derivative to determine where the function being investigated is increasing, together with a little common sense, a method much more enlightening and valuable to the student. So keen, indeed, is the author to use the second-derivative

test, that he anticipates it (in example 7.7) and has to refer forward to it. In example 7.6 (in which the least value of a variable G is being found) we are told that if G assumes a minimum value for some value of x , then for that value $D_x G = 0$, even though it is specifically pointed out earlier (p.98) that a function can have an extremum where it is not differentiable. So practice becomes separated from theory! And in all examples the author, though looking for greatest and least values, is content to find relative maxima and minima.

The author puts the reader very properly on his guard against certain ambiguities of notation and there are several places where he explains that a symbol has a double meaning; in particular he remarks that there is a tendency (a word which the reviewer feels could be replaced by a stronger one) to use ϕ and $\phi(x)$ interchangeably. On the integral notation he writes "It is not easy to avoid confusion over the double use of \int and dx . It would be far better if an entirely different symbol were used for the definite integral. But the use of these symbols for the two different purposes has become so entrenched in the literature that it takes more courage than the present author has to make a change". The reviewer suggests an alternative solution: leave the notation for definite integrals, and avoid (rather than replace the notation for) indefinite integrals. The author has gone some way towards this already in replacing systematic integration by systematic solution of separable first-order equations. The book is now in its third edition: if the author would like, in the fourth, to take one more step towards clearing up the confused notation of the elementary calculus, he will have this reviewer at least on his side.

H. A. Thurston, University of British Columbia

Mathematical Methods for Digital Computers, edited by Anthony Ralston and Herbert S. Wilf. Wiley, New York, 1960. 293 + xi pages. \$9.00.

This book should be in the hands of every person concerned with the choice or development of numerical methods for the digital computer solution of general scientific, including statistical, problems. Indeed, although it is not formally a textbook on numerical analysis, it is certain to give the mathematically literate reader, with the most rudimentary knowledge of the organization and operation of computing machinery, a clear grasp of the way in which computers are set up to solve mathematical problems, and some idea of the scope and limitations of existing techniques of numerical analysis.

The book contains 26 articles, by different authors, written, with a single exception, in a uniform format; each dealing with a different numerical technique. The articles include paragraphs on the formulation