EXTENDED ATMOSPHERES OF PLANETARY NUCLEI

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ABSTRACT

Exploratory calculations on nongray, hydrostatic-equilibrium model envelopes for central stars of planetary nebulae of high temperature and possibly near the instability limit are reported. It is conjectured that these may be related to Wolf-Rayet type nuclei; it appears possible to obtain an OV absorption, and OVI emission, spectrum even in an LTE calculation.

Key words: nongray, central stars of planetary nebulae, Wolf-Rayet nuclei, instability limit.

There are a number of theoretical and empirical arguments that indicate that central stars of planetary nebulae in the range 5.5×10^4 °K $< T_{eff} <$ 9.0×10^4 °K have extended atmospheres (in the sense that curvature effects become very important).

From a theoretical point of view we must realize that:

(1) Because of the high effective temperature of these objects, the scale height

$$H = RT/\mu g \qquad (1)$$

tends to be rather large; and

(2) The objects in the temperature range quoted are fairly close to the instability limit due to ra-

diation pressure effects (Harman and Seaton 1964) as defined by

$$|g| = |g_{rad}| = \frac{\pi}{c} \int_0^\infty \{\kappa_\nu + \sigma_{e\ell}\} F_\nu d_\nu \approx \frac{\sigma T_{eff}}{c} \sigma_{e\ell} \quad (2)$$

with g = surface gravity of the star, g_{rad} = radiative acceleration, κ_ν = monochromatic absorption coefficient, $\sigma_{e\ell}$ = electron scattering coefficient. This fact leads to a rather low effective surface gravity in these stars which in turn leads to an additional considerable increase of the scale heights. As pointed out earlier (Böhm 1969), one gets hydrostatic atmospheres with a thickness larger than one stellar radius if one approaches the instability limit (2) to a point where

$$|g_{rad}| \approx 0.9 \times |g| \quad . \quad (3)$$

In such an atmosphere curvature effects become very important. This point will be discussed below.

From an empirical point of view it is important to note (O'Dell 1968) that central stars showing Wolf-Rayet spectra occur practically only on the part of the Harman-Seaton sequence that lies close to the instability limit.*

Since Wolf-Rayet spectra can be formed only in a rather extended atmosphere (or envelope) we may also conclude that the observational evidence indicates that stars in this region of the T_{eff}-g-plane have extended envelopes. One might even hope that a thorough study of atmospheres in the indicated range of T_{eff}- and g-values will eventually lead to a the-

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* In view of the considerable uncertainties in the empirical determination of T_{eff} and g, it is difficult to be absolutely certain about a statement like this (Cf. diagram no. 7 in Böhm 1968). However, the evidence given by O'Dell (1968) certainly shows that the above statement at least does not contradict the present observational evidence.

oretical understanding of the Wolf-Rayet phenomenon.**

(A study of planetary nuclei has one great advantage as compared to the investigation of WR field stars: We know T_{eff} and g before we start the study.)

The problem of constructing model envelopes for central stars is being approached in three steps:

(1) in order to get some basic orientation, Böhm (1969) calculated some nongrey hydrostatic models near the instability limit using a plane-parallel approximation. Of course these results can be considered at best only as a qualitative approximation. However, even these crude calculations offer some interesting insight. They show the very strong dependence of the geometrical thickness of the atmosphere and also of the energy distribution of the surface flux $F_\nu(0)$ on g (Böhm 1969). In this connection it is also instructive to see how strongly the density stratification is changed by a rather small change in the surface gravity. This is illustrated by Figure 1.

(2) J. Cassinelli is trying to construct hydrostatic nongrey model envelopes for these objects taking into account curvature effects. He showed that a generalization of Lucy's (1964) temperature connection procedure is possible though the setting up of a relation between the zero-order moment J_ν and the second-order moment K_ν of the intensity $I_\nu(\mu)$ offers some difficulties in the spherically symmetric case. As in the plane-parallel case the determination of the monochromatic J_ν for a given $B_\nu(\tau_\nu, r)$ has to be carried out for every frequency between two successive temperature correction steps. This can be accomplished e.g. by the application of Carlson's S_n method (Cf. Carlson and Lathrop 1968). It is obvious that the determination of nongrey model envelopes is a laborious task (even if one makes the drastic simplification of assuming LTE). It is therefore important to ask whether the effort is worthwhile and what one may hope to accomplish. We do hope that a detailed study of these envelopes will eventually lead to an understanding of central star

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** As J. Schmid-Burgk (1968) has pointed out stars close to the instability limit (2) (but not yet in the actual instability region) easily generate a "strong stellar wind." This would make the mass loss of these Wolf-Rayet stars understandable.

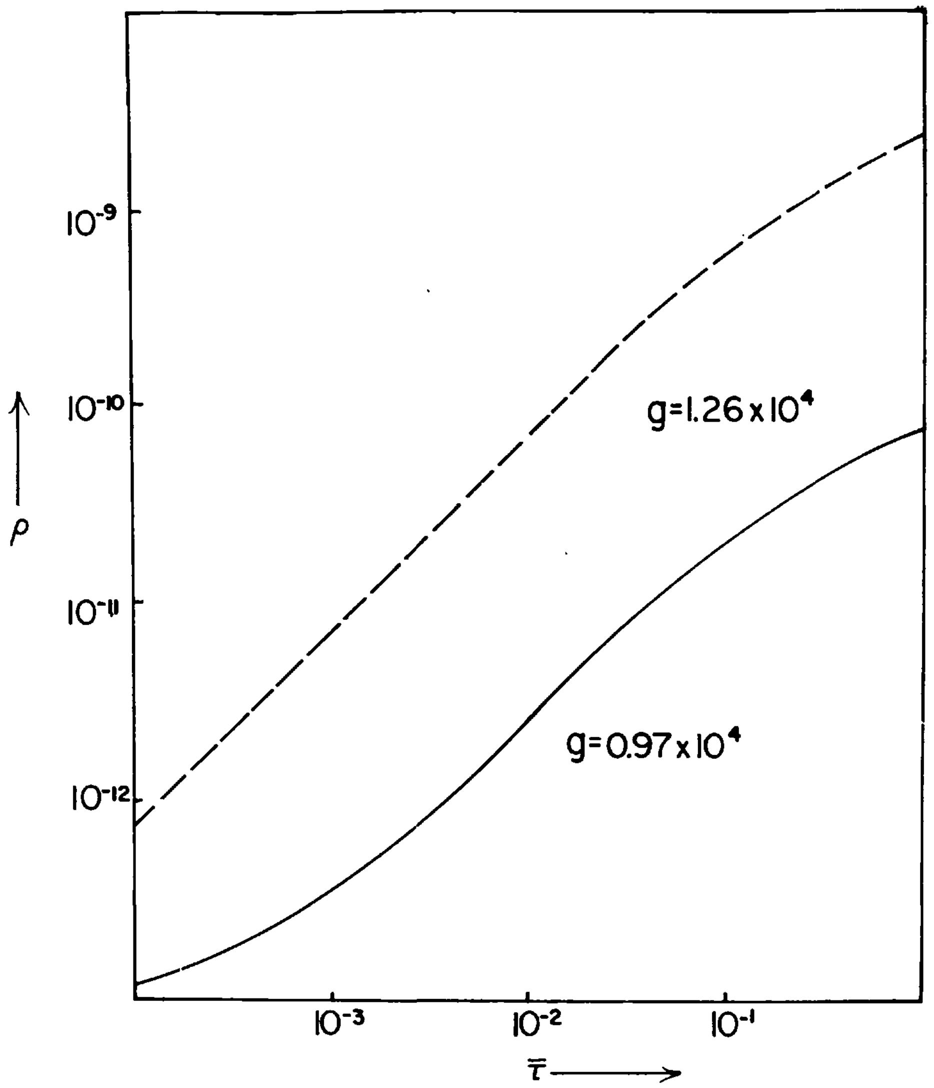
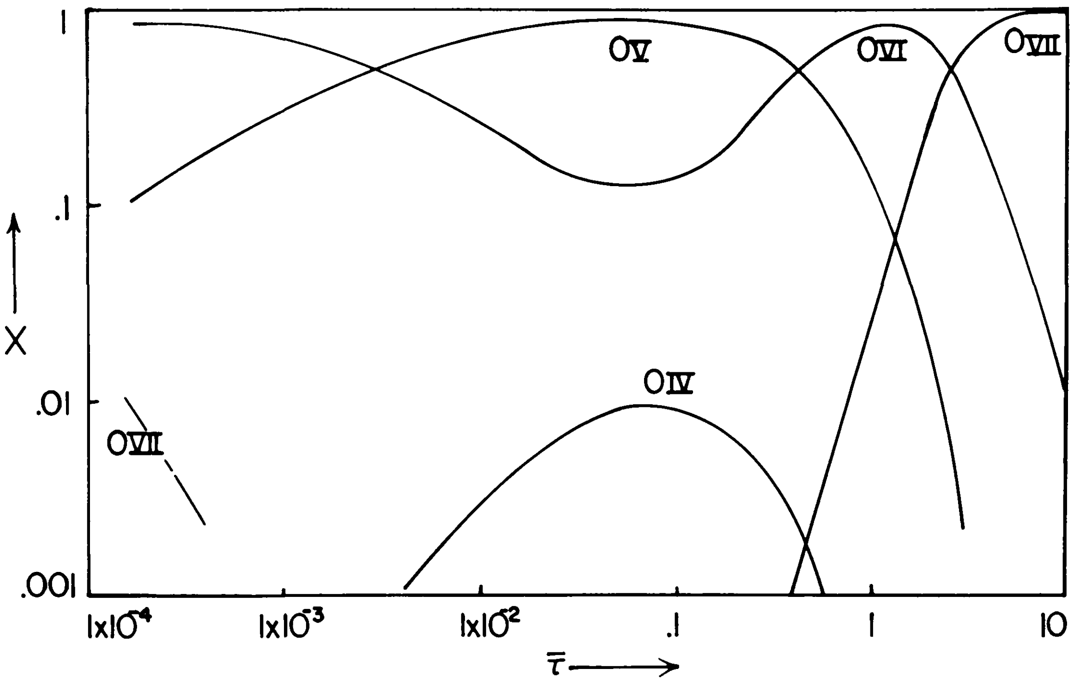


Figure 1. The density stratification $\rho(\bar{\tau})$ for the models with $T_{eff} = 6.3 \times 10^4$ °K, $g = 1.26 \times 10^4$ cm sec^{-2} and $T_{eff} = 6.3 \times 10^4$ °K, $g = 0.97 \times 10^4$ cm sec^{-2}.

spectra showing emission lines. Preliminary calculations indicate that it might, for instance, be possible to understand why certain central stars, especially of the type NGC 246 (see Greenstein and Minkowski 1964), show OVI lines in emission while the OV lines appear in absorption. Figure 2 shows that with a simple LTE assumption for the model with $T_{eff} = 6.3 \times 10^4$, $g = 0.97 \times 10^4$ cm sec^{-2} we can get a situation where in the outer very extended part of the atmosphere ($\tau < 3 \times 10^{-3}$) the oxygen exists mostly in the form of OVI whereas in deeper layers down to $\tau = 0.45$ the OV dominates. It is obvious that such an extended atmosphere could give rise to an OVI emission spectrum (because the OVI exists in an "extended envelope" that is optically thin in the continuous radiation) and an OV absorption spectrum.



igure 2. The ratio x of the number of ions in a
iven ionization stage to the total number of oxygen
ons as a function of τ for the envelope model with
$_{\rm eff}$ = 6.3 × 10^4 °K, g = 0.97 × 10^4 cm sec^{-2}.

hether this model could explain the observed spec-
rum in detail has not yet been checked. We felt
uch a detailed check would be worthwhile only after
really self-consistent spherically symmetric radi-
tive equilibrium model has been constructed.

(3) For the reasons indicated above it is ob-
ious that stellar envelopes with a continuous out-
low of matter and not in hydrostatic equilibrium
hould be investigated. Such a study is being car-
ied out by J. Schmid-Burgk. However, in this case
he combined hydrodynamics--radiative transfer prob-
em becomes so complex that one has to restrict one-
elf to a grey approximation of the radiative trans-
er problem. Curvature effects have been included
n this treatment.

We feel that the two lines of approach described
nder (2) and (3) should be followed simultaneously
n order to learn something about the nongrey radi-
tive transfer and the hydrodynamic aspects of the
roblem. One may hope, of course, that eventually
e shall be able to treat the whole problem combin-
ng the hydrodynamics and a realistic treatment of
he nongrey radiative transfer problem.

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DISCUSSION

Böhm: I would like to ask whether there are better methods to solve the nongrey spherical problem.

Hummer: There are papers by Mathis and Skumanich who treat these problems.

Rybicki: Perhaps the method of Feautrier can be generalized to solve the spherical problem.

Thomas: Are the shells you have calculated close to the instability limit?

Böhm: The observations show a very large scatter, so that it is not possible to find a clear decision. O'Dell, using new observations, does believe that for all planetary nebulae with central stars that are of the WR type, the shells are close to the limit of instability; but I think the scattering in absolute magnitude ($\Delta M = 1\overset{m}{.}5$) is too large for one to make this conclusion. Some of the central stars at the lower end of the sequence are Of stars.

Hummer: The nebulae that are connected with Of stars are very large in general. The calculated Zanstra temperatures do not correspond with the observed Of spectra.

Underhill: Some years ago I estimated roughly the temperatures of WR stars. I concluded from these temperatures that the difference between WC and WR stars is not due to differences in abundance but it is differences in excitation.

Wellman: I have tried to solve the transfer problem in spherical atmospheres by using the following procedure: The radiation is split up in two parts, (1) the radiation that is coming directly from the star, and (2) the diffuse radiation of the shell. If you integrate each part independently, you get a simple solution of the problem.

Böhm: That is the same procedure we followed, but we feel uncertain about it. In the nongrey case the radiation in different frequencies is coupled, and we should solve the general transport equation.