# THE APPLICATION OF RADIO INTERFEROMETRIC TECHNIQUES TO THE DETERMINATION OF EARTH ROTATION

Kenneth J. Johnston
E. O. Hulburt Center for Space Research
Naval Research Laboratory, Washington, D. C. 20375

Radio interferometric techniques for the determination of Earth rotation are described.

## INTRODUCTION

The effect of variations in the Earth's rotation and polar motion in radio interferometry is to alter the orientation of the baseline, i.e., the vector separation of two antennas receiving a signal from a radio source. This can easily be seen by the following. Consider a right handed coordinate frame with the X axis along the Greenwich meridian and the Z axis toward the Conventional International Origin (CIO) pole.

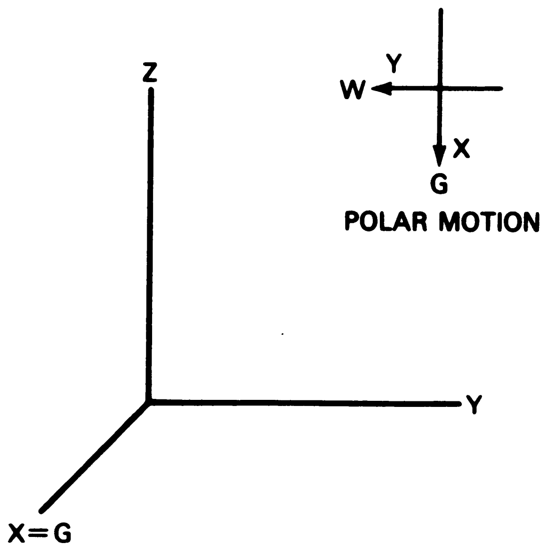


Figure 1. Right handed coordinate reference frame with X axis towards Greenwich and Z axis towards north.



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The baseline (X, Y, Z) joining the two antennas in topocentric coordinates will be rotated to

$$\begin{Bmatrix} X^1 \\ Y^1 \\ Z^1 \end{Bmatrix} = \begin{Bmatrix} 1 & +\alpha & -x \\ -\alpha & 1 & y \\ x & -y & 1 \end{Bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \qquad (1)$$

where $\alpha$ is the hour angle displacement of the equinox, and x and y are the polar coordinates relative to the CIO pole.

The change in baseline coordinates ($\Delta X$, $\Delta Y$, $\Delta Z$) may be expressed as

$$\Delta X = +\alpha Y - x Z \quad ,$$

$$\Delta Y = -\alpha X + y Z \quad , \qquad (2)$$

$$\Delta Z = x X - y Y \quad .$$

One can see that changes in UT1 effect only the equatorial baseline coordinates X and Y. Elsmore (1973) pointed out that an east-west baseline is not sensitive to the effects of polar motion to first order (see equation 2) and therefore UT1 can be measured directly using a single east-west baseline. To measure polar motion successfully, there must be a significant north-south component in the baseline length. With a significant north-south component in baseline length, it is impossible to measure all three components of the Earth's motion (UT1, x, and y) from a single baseline as is easily seen by inspection of equation 2. To measure all three components, it is necessary to use two independent baselines that preferably should be perpendicular. These changes in baseline coordiinates with respect to the instantaneous axis of rotation of the Earth together with the observed source's position on the sky define the interferometer phase, delay and fringe rate. For example, to first order the phase in radians is

$$\theta = 2\pi \ [Z \sin\delta + \cos\delta \ (X \cos\lambda + Y \sin\lambda)] \qquad (3)$$

where X, Y, and Z are now the baseline components in wavelengths in a right-handed coordinate frame where $\delta$ is the source declination and $\lambda$ is the source right ascension less the sidereal time at Greenwich. A change in phase, $\Delta\theta$, due to the effects of Earth rotation and polar motion may be expressed as

$$\Delta\theta = 2\pi \ [\Delta Z \sin\delta + \cos\delta \ (\Delta X \sin\lambda - \Delta Y \cos\lambda)], \qquad (4)$$

or

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$$\Delta\theta = 2\pi\ [+\ \alpha\ (-X \cos\delta \cos\lambda + Y \cos\delta \sin\lambda)$$

$$+\ x\ (X \sin\delta - Z \cos\delta \sin\lambda)$$

$$+\ y\ (-X \sin\delta - Z \cos\delta \sin\lambda)]\ . \qquad (5)$$

Since delay is the rate of change of phase with frequency and fringe rate is the rate of change of phase with time, similar relationships may be derived for these observables. Inspection of equation (4) shows that its time derivative has no dependence on the Z component of the baseline, and is therefore sensitive to changes only in the X and Y components of the baseline. For equatorial baselines, fringe rate data, as well as phase and delay data, can yield values of UT1 from a single baseline.

The sensitivity of radio interferometric measurements of Earth rotation and polar motion is dependent on the fractional accuracy to which the baseline length may be measured in the plane perpendicular to the axis of rotation around which the motion takes place; for UT1 it is the equatorial baseline length. For example, if a 3.6 km east-west baseline is used to achieve an accuracy of 4.3 ms in UT1 measurement (Elsmore 1973) the baseline length must be measured to an accuracy of 1.1 mm, while for a 3600 km east-west baseline, the corresponding accuracy is 1.1 m.

For accuracies approaching 1 ms in UT1 and 0".01 (30 cm) in polar motion, the baseline length must be measured quite accurately. The shorter baseline techniques employing common local oscillators have baselines ranging from 3.6 to 35 km. For the longest equatorial baselines used of approximately 20 km, this accuracy corresponds to 1.4 mm. For baseline lengths used in VLBI which are approximately 3500 km this accuracy is 25 cm.

## ACCURACY LIMITATIONS

There are several sources that contribute to inaccuracies in baseline measurement. The largest contributing error is caused by inaccuracies in knowledge of the differential path delay in the atmosphere of the radio signal between its arrival at the two antennas. The ionosphere will not be considered here because by making dual frequency measurements, the ionospheric delay may be calculated. It has an inverse frequency-squared dependence allowing it to be modeled accurately. For the atmosphere, the zenith path-length delay is approximately 2.3 meters for the dry atmosphere which is nearly constant, and between 1 and 25 cm at the zenith for the wet component. At 18 cm the zenith pathlength in the ionosphere and atmosphere are approximately equal but of opposite sign. The delay in the atmosphere may be modeled with ground based observations of temperature, pressure, and relative humidity. This may lead to an accuracy of ∿ 1-2 cm (Hopfield,1971) in determination of the zenith path delay at each site. Improved accuracy in measurement of the wet term may be achieved by using microwave radiometers to measure the column density of water vapor along the line of sight to the source. The accuracy of this method is hoped to approach 1 cm. These measurements are important for long baselines where the atmosphere at the antenna sites is largely independent. For shorter baselines less than 35 km, where the atmosphere may not be independent, the differential atmospheric phase path between the antennas may be more easily obtained through simple models. Figure 2 displays the rms phase fluctuations converted into path-length versus baseline length for baselines less than 35 km. These are short term fluctuations on the order of 10-15 minutes that may be averaged out with longer integration periods. Lack of knowledge of the exact differential path length of the signal in the atmosphere may contribute from < 1-3 mm error in the path-length for short baselines and several centimeters for longer baselines.

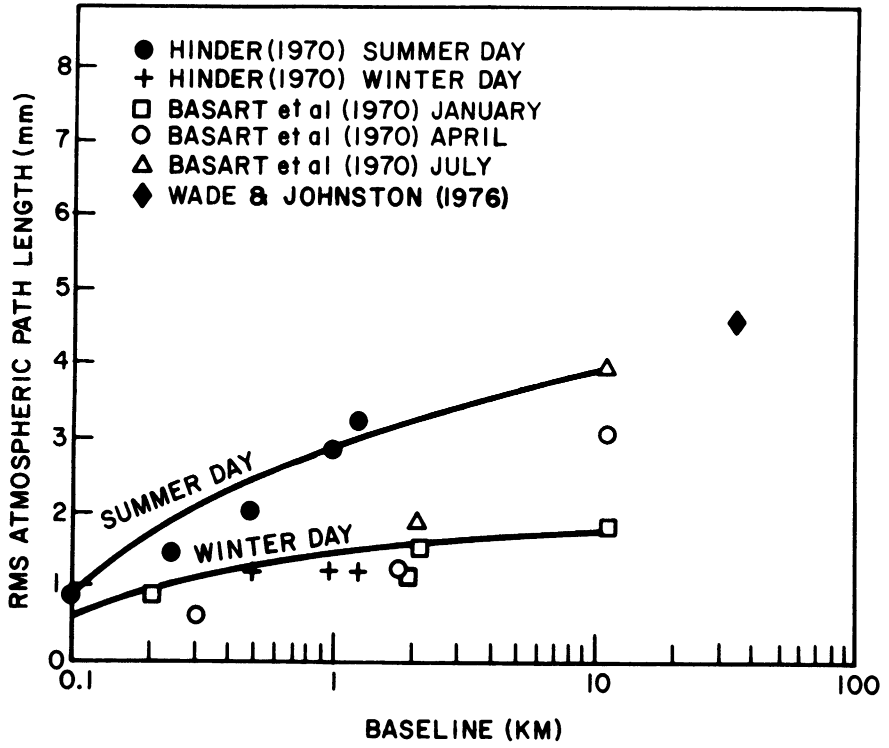


Figure 2. Variation of rms phase fluctuation versus baseline length for short baselines.

The shorter the baseline employed, the more the observations are subject to systematic effects that may cause small changes in baseline length. Deformation of the antenna structure and misalignment of the antenna antenna axis will lead to substantial systematic errors if they are not

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accounted for. The reference point or phase center for an antenna depends upon its construction. For an alt-azimuth antenna, the azimuth rotation plane and elevation axes usually intersect, making this the reference point of the antenna. The rotation axes on equatorially mounted antennas usually do not intersect. The reference point is along the hour angle axis where it orthogonally intersects a plane containing the declination axis. In calculation of the correlated interferometer phase, there is a term for the lack of axis intersection. For non-intersection of axes in the alt-azimuth case the term is k cos (el) while for the equatorial case it is k cos ($\delta$) where el is the source elevation, $\delta$ is the source declination, and k is the separation between the points where the axes should intersect. In addition to these axis offset terms, there are additional terms for axis misalignment. For an equatorial telescope the effect on phase is given by (Wade 1970)

$$\Delta\phi = -\ell \sin \delta \cos H - m \sin \delta \sin H, \tag{6}$$

where $\ell$ and m are the projection of the misalignment of the x and y axes, which are perpendicular to the polar axis of the telescope, $\delta$ is the source declination and H is the source hour angle. These terms, if not solved for, will give substantial errors in source position for the Green Bank interferometer where k = 60 wavelengths at 11.1 cm or approximately 0".5 on the 2.7 km baselines and approximately 0".05 for the 35 km baseline. For equatorial antennas these terms correlate with solutions for source position and baseline depending upon the selection of observed sources and the method of observation.

Antenna movements caused by deformations of the Earth's crust must also be taken into account. Earth tides cause a substantial motion in the individual antenna positions amounting to approximately 20 cm. Again, since this is a differential effect, for baselines, less than 35 km, it will amount to less than 1 mm while for transcontinental baselines it may reach about 40 cm. The Love numbers predicting the amplitude of this effect are known to 10%, putting this effect at the many centimeter level for the longer baselines. The amplitudes of the deformation now become variables to be solved for in the baseline solution.

The structure of radio sources may put a limitation on the determination of precise positions of these objects. Comparison of precise positions of radio sources determined by short and long baselines may not be valid without correction for source brightness distribution. Short-baseline (D < 10 km) observations at 6 cm refer to radiation from spatial sizes 1 arc second or less while longer baselines (D > 3000 km) at 6 cm refer to radiation having a size structure less than 0".004. The angular distribution of radio brightness of radio galaxies, Quasars and BL Lac objects is complex with component sizes ranging from about 0".01 to well under 0".001. Quasars and BL Lac objects have high intrinsic intensity and energy density. Variations in intensity and structure occur in a time scale of weeks to months which may have to be monitored in order to compare measurements made with different spatial resolution. These changes in structure may limit the comparison of positions made with different spatial resolution to the 0".01 level or even the stability of the position to this level. Thus far, at this level the positions appear to be coincident (Wade and Johnston, 1977). It is also assumed that since these objects are extragalactic, they will display no noticeable proper motion, i.e. 0".0001/yr. However variations in source structure over a period of years may shift the centroid of the radio brightness distribution substantially even at a single frequency and spatial resolution.

Relativistic effects caused by motions of the Earth-Moon system and by the gravitational field of the Sun must be accounted for and are at the 0".01 level.The values of astronomical constants which describe precession and nutation will have to be improved via radio observations. Daily observations to an accuracy of 0".01 will improve the short period nutation terms and as the observational time base approaches ten years, the value of the precession constant will be improved. These effects will be < 0".01. In summary these effects at present are at the millimeter level for short baseline lengths and five centimeter level for VLBI baseline lengths.

## REPORTED RESULTS

The results published in the literature thus far for short baselines are those of Elsmore (1972) who used a 3.6-km baseline over a nine month period to measure UT1 to an accuracy of 4.3 ms. For longer baselines, Shapiro et al. (1973) reported an accuracy of 2.9 ms in UT1 measurement with a baseline of 3899 km over a period of approximately one year. More recent observations will be reported later in this symposium.

There is a definite need for dedicated systems to measure Earth rotation parameters. Interested users will be applying this technique for the measurement of Earth rotation and polar motion. Only dedicated systems will make the daily or weekly measurements which will allow the evaluation of the true accuracy of this technique. These dedicated systems will be the topic of presentations being made by the National Geodetic Survey and the U.S. Naval Observatory/Naval Research Laboratory effort.

## A FUTURE NEW SYSTEM

New instruments are coming on line. One instrument which may be of interest, but is not discussed by others at this symposium, is the Very Large Array (VLA) of the National Radio Astronomy Observatory. The VLA in Socorro County, New Mexico, when completed in 1981, will have the ability to measure UT1 and both components of polar motion. This instrument consists of twenty-seven, 25-meter antennas which will be located on an equiangular "y" configuration, each arm 21 km in length with one arm of the "y" pointing slightly off a north-south line. The three antennas at the outermost points on the "y" will have baselines of

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approximately 36-km east-west and two 36-km baselines at a position angle of approximately ± 30°. This site is at an elevation of 7000 feet and is hopefully high enough above the majority of the atmopshere, so it will have a minimum effect on baseline accuracy. Displayed in Figure 3 are the interferometer phase of the strong radio sources 1749+701(*), 1642+690(o), DA426(▫), and NRA0512(▵) for observations over a 12 hour period for baseline lengths of 1-10 km for a frequency of 4880 MHz.

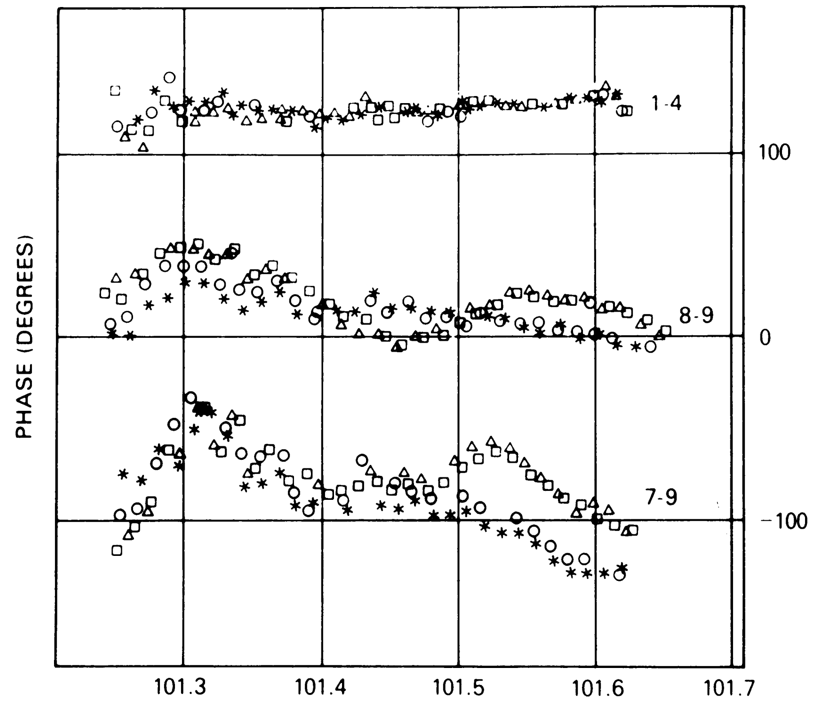


Figure 3. Correlated phase in degrees versus time in days for antenna baselines of 1 km (1-4), 2.5 km (8-9) and 10 km (7-9).

Unfortunately the instrumental phase stability for antenna 9, which was located at the 10 km spacing was poor, and no estimate of the rms fluctuation in phase of the atmosphere may be made. However this instrument when operational will allow the evaluation of atmospheric effects for baselines ranging from 1-36 km. At the present time there is information on the atmosphere for several sites for baselines of length $\leq$ 5.0 km. The Cambridge array has spacings to 5 km while the Green Bank radio link gives a single 35 km spacing.

## SUMMARY

Radio interferometry appears to offer an accurate method for the determination of Earth rotation parameters. Measurements made thus far are

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preliminary in nature and the accuracy of the technique can only be evaluated through daily observations over a long period of time. These observations are being planned or are in progress as you will hear from other papers reported at this meeting.

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## DISCUSSION

H. G. Walter: Do plans exist among the groups which perform radio interferometric measurements to agree on one or two standard sources to which right ascensions will be referred?

K. Johnston: There are no plans to do this. However, there are about 10 sources common to all catalogs now published; these sources can be used to relate the zero points of right ascension in these catalogs.

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