Introduction to Part II

Ever since 't Hooft [124] and Mandelstam [125] put forward the hypothesis of the dual Meissner effect to explain color confinement in non-Abelian gauge theories, people were trying to find a controllable approximation in which one could reliably demonstrate the occurrence of the dual Meissner effect in these theories. A breakthrough achievement was the Seiberg–Witten solution [2] of $\mathcal{N} = 2$ supersymmetric Yang–Mills theory. They found massless monopoles and, adding a small ($\mathcal{N} = 2$)-breaking deformation, proved that they condense creating strings carrying a chromoelectric flux. It was a great success in qualitative understanding of color confinement.

A more careful examination shows, however, that details of the Seiberg–Witten confinement are quite different from those we expect in QCD-like theories. Indeed, a crucial aspect of Ref. [2] is that the SU(N) gauge symmetry is first broken, at a high scale, down to $\mathrm{U}(1)^{N-1}$, which is then completely broken at a much lower scale where condensation of magnetic monopoles occurs. Correspondingly, the strings in the Seiberg–Witten solution are, in fact, Abelian strings [36] of the Abrikosov–Nielsen–Olesen (ANO) type which results, in turn, in confinement whose structure does not resemble at all that of QCD.[1] In particular, the "hadronic" spectrum is much richer than that in QCD [126, 127, 128, 35, 129]. To see this it is sufficient to observe that, given the low-energy gauge group $\mathrm{U}(1)^{N-1}$, one has $N - 1$ Abelian strings associated with each of the $N - 1$ Abelian factors. Since

$$\pi_1\big(U(1)^{N-1}\big) = Z^{N-1}, \tag{3.5.38}$$

[1] It is believed, however, that the transition from this Abelian to non-Abelian confinement is smooth. The Seiberg–Witten construction has an adjustable parameter μ which governs the strength of the ($\mathcal{N} = 2$)-breaking deformation. In passing from the Abelian strings at small μ to non-Abelian at large μ no phase transition is expected. One cannot *prove* this statement at present because the large-μ side is inaccessible. In contrast, in the models to be discussed below, we can address and explore *directly* non-Abelian strings and associated dynamics.

the Abelian strings and, therefore, the meson spectrum come in $N-1$ infinite towers. This feature is not expected in real-world QCD. Moreover, there is no experimental indication of dynamical Abelization in QCD.

Here in Part II we begin our long journey which covers the advances of the last decade. Most developments are even fresher, they refer to the last five years or so. First we dwell on the recent discovery of non-Abelian strings [130, 131, 132, 133] which appear in certain regimes in $\mathcal{N}=2$ supersymmetric gauge theories. Moreover, they were found even in $\mathcal{N}=1$ theories, the so-called M model, see Section 5.2. The most important feature of these strings is that they acquire orientational zero modes associated with rotations of their color flux inside a non-Abelian $\mathrm{SU}(N)$ subgroup of the gauge group. The occurrence of these zero modes makes these strings non-Abelian.

The flux tubes in non-Abelian theories at weak coupling were studied in the past in numerous papers [134, 135, 136, 137, 138, 139, 140]. These strings are referred to as Z_N strings because they are related to the center of the gauge group $\mathrm{SU}(N)$. Consider, say, the $\mathrm{SU}(N)$ gauge theory with a few (more than one) scalar fields in the adjoint representation. Suppose the adjoint scalars condense in such a way that the $\mathrm{SU}(N)$ gauge group is broken down to its center Z_N. Then string solutions are classified according to

$$\pi_1\left(\frac{\mathrm{SU}(N)}{Z_N}\right)=Z_N.$$

In all these previous constructions [134, 135, 136, 137, 138, 139, 140] of the Z_N strings the flux was always directed in a fixed group direction (corresponding to a Cartan subalgebra), and no moduli that would allow to freely rotate its orientation in the group space were ever obtained. Therefore it is reasonable to call these Z_N strings Abelian, in contrast with the non-Abelian strings, to be discussed below, which have orientational moduli.

Consideration of non-Abelian strings naturally leads us to confined non-Abelian monopoles. We follow the fate of the classical 't Hooft–Polyakov monopole (classical not in the sense of "non-quantum" but rather in the sense of something belonging to textbooks) in the Higgs "medium" – from free monopoles, through a weakly confined regime, to a highly quantum regime in which confined monopoles manifest themselves as kinks in the low-energy theory on the string world sheet. The confined monopoles are sources (sinks) to which the magnetic flux tubes are attached. We demonstrate that they are dual to quarks just in the same vein as the magnetic flux tubes are dual to the electric ones.

Our treatise covers the non-Abelian flux tubes both in theories with the minimal ($\mathcal{N}=1$) and extended ($\mathcal{N}=2$) supersymmetry. The world sheet theory for the Abelian strings contains only translational and supertranslational moduli fields.

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At the same time, for the non-Abelian strings the world sheet theory acquires additional massless (or very light) fields. Correspondingly, the Lüscher term coefficient, which counts the number of such degrees of freedom, changes [141].

The third element of the big picture which we explore is the wall-string junction. From string/D brane theory it is well known that fundamental strings can end on the brane. In fact, this is a defining property of the brane. Since our task is to reveal in gauge theories all phenomena described by string/D brane theory we must be able to see the string-wall junctions. And we do see them! The string-wall junctions which later got the name *boojums* were first observed in an $\mathcal{N} = 2$ gauge theory in Ref. [142]. This construction as well as later advances in the boojum theory are thoroughly discussed in Part II.

The domain walls as brane prototypes must possess another remarkable feature – they must localize gauge fields. This localization was first proven to occur in an $\mathcal{N} = 2$ gauge theory in Ref. [37]. The domain-wall world sheet theory is the theory of three-dimensional gauge fields after all!

All the above elements combined together lead us to a thorough understanding of the Meissner effect in non-Abelian theories. To understand QCD we need to develop a model of the *dual* Meissner effect. Although this problem is not yet fully solved, we report here on significant progress in this direction.