uniformly in n, for polynomials whose coefficients are $n \times n$ matrices. Such a T is said to be completely polynomially bounded. Since this book went to press, Pisier has obtained a negative answer to Q3 by constructing a polynomially bounded operator which is not completely polynomially bounded. His proof depends upon a deep multiplier theorem which extends the ideas of Chapter 6.

It is often difficult to check whether a given map is completely bounded. A frequently used tool is Grothendieck's Inequality, which is introduced in Chapter 5, together with some of its variants. One formulation is to say that the only bounded Schur multipliers $M_\varphi : [a_{ij}] \mapsto [\varphi(i,j)a_{ij}]$ of $B(H)$ are given by elements of the projective tensor product $\varphi(i,j) \in \ell_i^\infty \hat{\otimes} \ell_j^\infty$. Pisier obtained a non-commutative version of this result in which ℓ^∞ is replaced by any C^*-algebra A. In Chapter 7 this is used to verify significant special cases of Q2 such as Haagerup's Theorem. This asserts that if $u : A \mapsto B(H)$ has a cyclic vector ξ with $\{u(a)\xi : a \in A\}$ dense in H, then there is an invertible $S \in B(H)$ for which $a \mapsto S^{-1}u(a)S$ is a $*$-representation.

Chapter 8 features a discussion of completely bounded maps in the Banach space setting. The results are in the spirit of the author's earlier work on factorization of linear operators.

Despite the wealth of ideas and results presented in this short book, the text is not hard going and makes a good read. The author does not embark upon proofs requiring heavy harmonic analysis, although several are cited amongst two hundred references. This volume realises the author's objective of being a suitable basis for an advanced graduate course in functional analysis and may be strongly recommended. Several problems in this area remain to be solved.

G. BLOWER

ADAMS, D. R. and HEDBERG, L. I. *Function spaces and potential theory* (Grundlehren der mathematischen Wissenschaften, Band 314, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1996), xi + 366 pp., 3 540 57060 8, (hardcover) DM148.

A great deal of mediocre mathematics goes under the name of *function spaces*. This book, I am very pleased to say, is not about such mathematics. Indeed, it is an exposition of contemporary potential theory at the highest level, given by two of the foremost presences in the subject. It is in the honourable tradition of Gauss and later Frostman and is much influenced by Carleson [2] and Maz'ya.

The natural language for modern potential theory, as for much of modern analysis, is that of function spaces – principally the Lebesgue and Sobolev spaces and their variants. But the spaces are not the central objects of study in this book – merely the vehicle for the correct formulation of potential theory. (I once examined what was a very good Ph.D. thesis on function spaces; the student had developed a number of highly original and beautiful ideas. As a mere formality I asked him to explain why the Triebel–Lizorkin space $F_0^{p,2}$ coincided with the Lebesgue space L^p. He had no idea. He was awarded the degree.)

The point of view of the book under review is that (as is well-known) in the study of nonlinear problems in PDE it is often the case that the most important information is contained in L^p-based spaces rather than just L^2-based ones. Thus potential theory based upon say $\int |\nabla u|^p dx$ rather than $\int |\nabla u|^2 dx$ is required. This leads to corresponding potential operators which now have the form $\mu \mapsto G_\alpha * (G_\alpha * \mu)^{p'-1}$ (where G_α is, say, a standard Bessel potential kernel) and which are manifestly nonlinear when $p \neq 2$. (Of course, $1/p + 1/p' = 1$.) It is the study of such operators, their corresponding capacities and the function spaces upon which they act that forms the core of this book and as such there is relatively little overlap with other leading books in the field such as [1] or [3]. It turns out that the action on the so-called Besov and Triebel–Lizorkin spaces $B_\alpha^{p,q}$ and $F_\alpha^{p,q}$ is both nontrivial and unexpected, hence the prominence of the function spaces in the title. Indeed, consideration of these spaces is essential even to understand the action on the more classical Sobolev spaces. Work of Frazier and Jawerth and of Netrusov (the latter's influence pervades the entire book) on the atomic approach to $B_\alpha^{p,q}$ and $F_\alpha^{p,q}$ turns out

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to be crucial here.

A brief description of the contents of the book (which is written for those who know real analysis but not necessarily potential theory) is as follows. Chapter 1 is concerned with preliminaries from real analysis (maximal functions, Fourier transform, etc.) and Sobolev spaces. Chapter 2 introduces the L^p-capacity and nonlinear potentials referred to above. Chapter 3 is a compendium of real variable estimates for Bessel and Riesz potentials which prove useful later. In Chapter 4 the Besov and Triebel–Lizorkin spaces are introduced and their role in potential theory established via a fundamental inequality of Tom Wolff. The next four chapters explore the consequences of what has been set up. Chapter 5 is a very useful guide to comparison of capacities with the (more familiar?) Hausdorff measures, while continuity properties of functions in Sobolev spaces – "continuous except on a 'small' set measured in terms of capacity" – are considered in Chapter 6. Chapters 7 and 8 treat trace and embedding theorems and Poincaré type inequalities respectively. Finally, Chapter 9 is concerned with a certain approximation property of Sobolev spaces and in Chapter 10 this property is re-examined in the light of Netrusov's ideas and a further theorem of Netrusov is presented. The last chapter deals with rational and harmonic approximation.

The book is carefully and thoroughly written and prepared with, in my opinion, just the right amount of detail included. The prose flows naturally. A couple of topics which might have been treated at least to some extent are irregular domains (of vast importance in potential theory) and trace and embedding theorems between function spaces when the target space also measures smoothness. But these are perhaps just the minor grumbles of a harmonic analyst – the book addresses a much wider audience than just the harmonic analysts – and indeed it is pleasing to note just how indispensable the Calderón-Zygmund theory is in the whole development of the subject.

Function spaces and potential theory will certainly be a primary source that I shall turn to when I need to consider potential-theoretic matters and, I suspect, it will be for many others too.

A. CARBERY

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HOWIE, J. M. *Fundamentals of semigroup theory* (London Mathematical Society Monographs No. 12, Clarendon Press, Oxford, 1995), x + 351 pp., 0 19 851194 9, (hardback) £45.00.

John Howie's latest book is a substantial updating of his 1976 book *An introduction to semigroup theory* (Academic Press). Like its predecessor the new book does not attempt to cover the whole field, but concentrates instead on the algebraic theory with a particular emphasis on the class of regular semigroups. Regular semigroups are easier to handle than arbitrary semigroups but, more importantly, they play a paradigmatic role in semigroup theory as a whole.

The book is divided into eight chapters. The first three might well have been subtitled "What every mathematician should know about semigroups". In the first chapter basic definitions from algebra are introduced with the minimum of fuss, including a useful treatment of equivalence relations and congruences. Semigroup theory proper begins with Chapter 2 and the definition of Green's equivalence relations. With the help of these relations the elements of a semigroup can be sorted according to their mutual divisibility properties and arranged in what is termed an "egg-box diagram". The properties of these diagrams are basic to many arguments in semigroup theory. The third chapter is an exposition of the Rees Theorem, one of the most influential results in semigroup theory. This result describes in a way reminiscent of the Wedderburn–Artin Theorem the

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