

detailed reading of a small (non-random) sample of the book the reviewer found a rather large number of misprinted symbols which are correctly printed in the Russian edition.

F. F. BONSALL

F. M. ARSCOTT, *Periodic Differential Equations* (Pergamon Press, 1964), vii + 275 pp., 602.

This book contains an account of a class of linear ordinary differential equations and the special functions generated by them. The author explains in the Preface that he has concentrated on fundamental problems and techniques of solution rather than the properties of particular functions. He has attempted to steer a middle course between books written primarily for engineers and those demanding a knowledge of advanced pure mathematics on the part of the reader. At the same time, he has tried to enable the reader to pass on to a more detailed study of either of the two types of books. The reviewer feels that the author succeeded in the task he set himself and produced a very useful and readable book. He is to be commended for the care with which the book was written; for using as far as possible the notations used in the books to which the reader might proceed from his; for providing ample documentation and references; and for greatly increasing the information contained in his book by the device of adding "examples" (i.e. mostly results from the literature not discussed in detail in the present book) to each chapter. There was no attempt at encyclopaedic completeness, and in spite of the considerable amount of material, the book makes an uncluttered impression.

After an introductory chapter, mostly on the origin of the various differential equations treated in this book, about one-half of the book is devoted to Mathieu's equation,

$$w'' + (a - 2q \cos 2z)w = 0.$$

Both the "general" equation (i.e., the equation with arbitrary given values of a and q) and Mathieu functions (the solutions for characteristic values of a for which a periodic solution exists) are treated.

In the second half of the book, one finds a variety of differential equations and their solutions: Hill's equation, the spheroidal wave equation, the differential equation satisfied by Lamé polynomials, and the ellipsoidal wave equation. A. ERDÉLYI

NAIMARK, M. A., *Linear Representations of the Lorentz Group*, translated from the Russian by Ann Swinfen and O. J. Marstrand (Pergamon Press, 1964), pp. xiv + 450, 100s.

One's first reaction on opening this book is to marvel that it is possible to write a book of four hundred and fifty pages on the representations of the Lorentz group, particularly so since this volume is not concerned with the generalised Lorentz group of r spatial and s temporal dimensions but only with the group of Lorentz transformations of the 3+1 space-time world. The full Lorentz group \mathcal{G} (called the general Lorentz group by the author) is the set of all real linear transformations $x'_i = \sum_j g_{ij} x_j$ which leave $x_1^2 + x_2^2 + x_3^2 - x_4^2$ invariant. This group consists of four disjoint pieces, the most important of which from the physical point of view is the subgroup called the proper Lorentz group \mathcal{G}_+ whose transformations satisfy $|g_{ij}| = +1$, $g_{44} \geq 1$. The proper Lorentz group \mathcal{G}_+ is a normal subgroup of index 2 of the complete Lorentz group \mathcal{G}_0 whose transformations are required to satisfy the less stringent condition $g_{44} \geq 1$ and amongst which are spatial reflections such as $x'_1 = -x_1$, $x'_2 = x_2$, $x'_3 = x_3$, $x'_4 = x_4$ for which $|g_{ij}| = -1$. \mathcal{G}_0 is in turn

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a normal subgroup of \mathcal{G} of index 2. For the remaining coset of \mathcal{G}_0 in \mathcal{G} and for the two remaining cosets of \mathcal{G}_+ in \mathcal{G} we have $g_{44} \leq -1$, corresponding to temporal reflections or a reversal of time. The present work has as its main object the description of all completely irreducible representations of \mathcal{G}_+ and \mathcal{G}_0 and the derivation and interpretation of the corresponding invariant equations. In so far as the representations by matrices of finite dimensions are concerned, the results are well known and, in the reviewer's opinion, more lucidly presented in works such as H. Boerner's *Representation of Groups*, but the large sections of the present work devoted to representations by bounded linear operators in a Banach space have not hitherto been easily accessible in writings in the English language. For this reason Professor Naimark's book is a welcome addition to the literature. The study of these infinite-dimensional representations may, the author suggests, prove as useful for the further development of quantum theory as have the finite dimensional representations in the elucidation of the concept of spin. Furthermore, they afford an admirable introduction to the general theory of infinite-dimensional representations of semi-simple Lie groups.

With physicists in mind the author has endeavoured to make the exposition as self-contained as possible. This approach, however, has its drawbacks from the point of view of the mathematician because the mathematical details are not presented in their wider context but only in their relation to the Lorentz group. For instance, page 1 is devoted to defining a group and on page 2 a footnote points out that only those facts concerning groups will be mentioned which are later required. Eventually after 88 pages, which include a full treatment of the representations of the rotation group, the concept of a subgroup is introduced by way of another footnote. The pure mathematician is likely to find this method of presentation tedious. It may commend itself, however, to the physicist provided he has the necessary stamina to absorb all the technical details exhibited. Basically, the difficulty is that it is doubtful whether any author can write a really satisfactory book for a reader assumed to be ignorant of groups, linear spaces, eigenvalues, Banach spaces, bounded operators, Hilbert spaces, residue classes and the like, which at the same time embodies a substantial amount of the author's own researches on unitary and other linear representations of the orthogonal and Lorentz groups. To fulfil such conditions is a very formidable task and the author has made a commendable attempt to accomplish it.

D. E. RUTHERFORD

FUCHS, B. A., AND SHABAT, B. V., *Functions of a Complex Variable and Some of their Applications*, Vol. I, original translation by J. Berry, revised and expanded by J. W. Reed (Pergamon Press, 1964), 431 pp., 70s.

This book has been written primarily for students of engineering and technology but, containing as it does a wealth of worked examples and an adequate number of exercises with solutions and hints, it could be valuable as a supplementary textbook for honours students in pure mathematics, who would find it easy to read and most illuminating. The subjects considered in the book are mainly those of any first course on complex variable theory but, as would be expected in a book written for applied scientists, considerable emphasis is placed on topics such as conformal mapping and harmonic functions. The treatment is clear, although not modern; complex numbers are treated as vectors, the proof given of Cauchy's theorem is the one using Green's theorem, and several theorems such as that of Morera and the open mapping theorem are quoted without proof. The book is a mine of useful information and can be read with profit by students of complex variable theory at all levels of sophistication.

D. MARTIN