Ergod. Th. & Dynam. Sys. (1989), **9**, 629–641
Printed in Great Britain

Vladimir Abramovich Rokhlin— A biographical tribute (23.8.1919–3.12.1984)

A. M. VERSHIK
Department of Mathematics, Leningrad University, USSR

Vladimir Abramovich Rokhlin was born on 23 August 1919 in Baku (Azerbaijan). His parents, Abram Beniaminovich Rokhlin and Henrietta Emmanuilovna Levenson, came from Jewish families, who lived in the Ukraine and in Byelorussia and then moved to Baku. Rokhlin's mother was the sister of the well-known literary figure and children's writer Kornei Chukovsky; Rokhlin's grandmother, Klara Levenson, was one of the first women doctors in Russia. Rokhlin's mother graduated from a medical school in France and was a doctor in Baku. She died tragically in 1923. His father was a broadly educated man and took an active part in political activity before the revolution (he was a social democrat) and in the early years of the revolution. Later he was involved in administrative work in Baku, in the Ukraine, in Central Asia and in Moscow. Not surprisingly he did not escape the Stalinist repressions: in 1939 he was arrested and on 13.7.1941 was sentenced to be shot. In 1957 his relatives received a certificate of rehabilitation ('the case is closed due to insufficient evidence'); it is clear from this certificate that it was still impossible to obtain reliable information about the last years of his life; in particular even the exact date of his death is not precisely established. Rokhlin's family was exiled to Siberia and remained there. Fortunately Rokhlin, who was at that time a student at the University of Moscow, escaped with comparatively minor unpleasantnesses, and was not expelled from the University.

The milieu of Rokhlin's family was characteristic of cultured Russian homes: interest in languages, literature, music; however it is not reported that anyone showed a special interest in mathematics.

At the age of 16, having skipped a class, Rokhlin graduated from secondary school in Alma-Ata (Central Asia). Alma-Ata was during those years a centre for political exiles, and at that time the standard of culture and, in particular, the organization of education in the school, where Rokhlin studied, were far better than what was typically provided away from major cultural centres. The docent at the pedagogical institute, M. Ustimenko, was Rokhlin's mathematics teacher at the school and was apparently an expert in his field. Here is what he wrote in Rokhlin's graduation report: 'Rokhlin possesses great mathematical abilities. He grasps the presented material extraordinarily quickly and easily, thinking it out critically and independently verifying a whole chain of logical conclusions ... Rokhlin's interest in mathematics is so strong that he independently studied the beginnings of calculus, analytical geometry and higher algebra. He also shows great interest in the history of mathematics. On the strength of what I have set forth, I am persuaded that Rokhlin will be a serious researcher in the field of mathematics, ..., that the path of investigation, and of research is appropriate for him.' Rokhlin fully justified his perspicacious teacher's evaluation of him.

At the age of not quite 16 Rokhlin enrolled in the Faculty of Mechanics and Mathematics at Moscow University; special permission from the then People's Commissar (Minister) of Education was needed for him to enrol at such an early age. He studied with enthusiasm. In the opinion of fellow students and instructors, he was an outstanding student, one of the strongest in the faculty. He attended special courses by Petrovski, P. S. Aleksandrov, Men'shov, Kurosh, Bari, Kolmogorov, Rashevski, Pontryagin, Plesner and others. He was interested in physics and attended lectures by Landsberg and later by Landau in the Faculty of Physics. He took 17 elective courses and special seminars in function theory, algebra, functional analysis, topology, algebraic geometry, etc.

His first advisor in his undergraduate years was Abram Iezikilovich Plesner (1900–1961). Plesner emigrated to the USSR from Germany in the early 1930s and was a professor at the University of Moscow from 1932 to 1948. He acquainted Moscow mathematicians with some subjects that were popular in the West at that time but were little known in Moscow: algebraic geometry, ergodic theory and in particular the works of Von Neumann and operator theory. These ideas found a response from the then very strong Moscow mathematical school. In particular Plesner directed Rokhlin into work on the development of these ideas. Rokhlin refined and published a long course of lectures by Plesner on the spectral theory of operators, which appeared in two long articles in 'Uspehi matematicheskikh nauk' in 1941 and 1946. These surveys were the first on this topic in the Russian literature; they enjoyed great popularity and helped many specialists, in particular in the theory of dynamical systems. Here is how Plesner characterized Rokhlin's work: 'Already in his undergraduate years Rokhlin revealed outstanding mathematical abilities and genuine scientific enthusiasm ... it was only thanks to his competent help that it was possible to prepare the whole article in a comparatively short space of time.' This was said about a fourth-year undergraduate. Later Plesner wrote, 'Possessing scientific initiative, Rokhlin, even at the very beginning of his undergraduate years, took part in research work, which was initially in topology and later in the spectral theory of dynamical systems. Having obtained significant scientific results in the study of the structure of dynamical systems, he continued his promising researches after becoming a postgraduate student, until the day when war tore him from studies.'

Taking part in P. S. Aleksandrov's famous seminar, Rokhlin completed two papers on set-theoretic topology, which were published in a collection of student papers. His diploma paper, 'Unitary rings and dynamical systems', as Kolmogorov wrote, 'is a scientific study of a standard required of a candidate's (PhD) dissertation'; it received the first prize at the student papers competition in 1940. At the same time, he proved a number of results about automorphisms of compact Abelian groups. Moreover, he prepared a survey, written on Pontryagin's initiative, but only published after the war – 'Homotopy groups' – almost the first survey of this subject. Kolmogorov later noted: 'These scientific-literary papers show Rokhlin to be an exceptional scholar and mature mathematician'. Later Kolmogorov and Pontryagin wrote in their letter, of which we shall speak below: 'Rokhlin seems to be the most outstanding and gifted of the young mathematicians of Moscow, doing their postgraduate work at the moment when the war began. We as professors were struck by Rokhlin's exceptional capacity for work and great power of thought concentrated on solving difficult problems.' Several chairs of the Mechanics and Mathematics Faculty nominated Rokhlin for a named stipend (the so-called Stalin), which he lost after his father's arrest, and almost all the chairs recommended him for postgraduate work. In 1940 Rokhlin graduated from the faculty and enrolled as a postgraduate student at the Institute of Mathematics and Mechanics at Moscow University. With Plesner as his supervisor and measure theory and dynamical systems as his field of study, Rokhlin kept close contact with Kolmogorov and Pontryagin. His prize-winning student work which formed the basis of the well-known paper, 'On the fundamental ideas of measure theory', published after the war, was written under Kolmogorov's influence, as a continuation of the latter's classical papers on the foundations of probability theory and was mostly devoted to the most subtle problem of the theory: conditional measures (the canonical system of measures of a measurable partition). This work was interrupted by the war.

At the beginning of July 1941 Rokhlin went as a volunteer to the front with the Moscow People's Volunteer Corps and enlisted in the 8th Red Presnya (a district of Moscow) Division, and afterwards as a private in the 995th Artillery Regiment. The Volunteer Corps was not at the beginning a part of the regular army; it consisted of essentially untrained and almost unarmed civilians, among whom were people absolutely unfit for service. It is not surprising that the enthusiasm and patriotic impulses of millions of people were exploited to cover the country's absolute lack of preparation for war and at all costs to maintain the army which was at that time falling to pieces. Tens of thousands of vain sacrifices, among whom were talented young people, the hope of our country, perished, paying with their lives for Stalin's criminal policy before and during the war. At the beginning of October, Rokhlin's unit was surrounded by the Germans in the region of Vyaz'ma (near Moscow): he was wounded in both legs and was left in the countryside in civilian clothes in the care of local residents. He attempted several times to cross the front and return to the Army, but without success. At the end of 1941 his wounds opened and he was put in the civilian hospital in the territory, occupied by Germans, and remained there until July 1942, when as the result of a denunciation and after a German trial he was sent to a prisoner-of-war camp. There he became ill with typhus, recovered, and was transferred to various camps in the territories of Byelorussia and Poland; he again tried to escape. In the camp he passed for a Baltic German. (Rokhlin knew German well; he had neither a Jewish accent nor any other incontrovertible signs of being Jewish.) He remained in the camp, thinking about mathematics, perfecting his German and survived despite new denunciations; at the beginning of 1945 he escaped and found himself at the rallying point of the 5th Army of the First Byelorussian Front and finished the war in Germany in the ranks of the Soviet troops. But as early as May 1945 he was sent to a Soviet deportation camp for former prisoners of war, and in January 1946 from there to a special examining camp at the settlement of Vozhael, in the Komi Autonomous Soviet Socialist Republic. In June of that year he was cleared, but ... was kept in the camp as a guard. The organization of tens of camps for former Soviet prisoners of war and displaced persons who had been in the territory occupied by the Germans, continued the series of Stalinist crimes. On the pretext of checking on people's activities during the war and disclosing 'hidden enemies', the humiliation of those who had drunk their full measure of the horrors of war or were witness to the failures of Soviet military and other politics was continued. Not the least role, obviously, was played by the desire to silence people about their experience beyond the borders of their country.

Nobody knows how long Rokhlin's camp saga would have lasted, but he managed to communicate information about himself to Moscow and in February 1946 Kolmogorov and Pontryagin appealed to Kruglov, the then People's Commissar (Minister) of Internal Affairs, and General Golikov, the Director of Repatriation Affairs (the latter is now generally mentioned in the press as an accomplice in many crimes) in a letter, which in particular contained the above-mentioned excellent evaluation of Rokhlin and also the following: 'We beg your attention to the fate of Vladimir Abramovich Rokhlin, who returned from a German prison camp and is at present in the Komi Autonomous Soviet Socialist Republic. ... Some first-class completed scientific studies are already due to him ... Judging by the beginning of his scientific career he promises to become an outstanding scholar; his return to scientific work would undoubtedly lead within the next few years to his obtaining significant new scientific results, ... therefore we consider that in the interests of the development of Soviet mathematics it would be extremely desirable to permit V. A. Rokhlin to return in as short a time as possible to his graduate studies in order to continue his scientific work under our supervision.' (February 13th, 1946.) Almost a year later, on December 4th, 1946, there followed a letter from the Director of the Administration for the Protection and Regime of the Main Administration of Corrective Labour Camps and Colonies (GULAG), addressed to V. V. Stepanov, the Director of the Institute of Mathematics and Mechanics of Moscow University: 'The GULAG of the Ministry of Internal Affairs of the U.S.S.R. has given the order to relieve V. A. Rokhlin from his duties as a guard and that he be placed at your disposal', and also that same director's command, sent to Ust'Vym' Corrective Labour Camp, for the official registration of his 'final discharge and assignment to the City of Moscow at the disposal of the Mathematical Institute'. In December 1946 Rokhlin returned to Moscow.

As early as June 1947 Rokhlin successfully passed his postgraduate examinations, and in December he brilliantly defended his candidate's dissertation on the topic, 'Lebesgue spaces and their automorphisms'; his official opponent (referee) was

Kolmogorov. Readers will acquaint themselves, not without interest, with his review:

'The latest development of the metric theory of dynamical systems has led to the necessity for a certain revision of measure theory. It turned out that in the finer questions it is natural to assume somewhat more of a measure space than was done earlier. These natural additional requirements are introduced by the author as separability and completeness. The requirement of separability is fulfilled in the majority of important special cases and does not much restrict the significance of Rokhlin's study, though an analogous study of non-separable spaces would also be of some interest. Rokhlin's key achievement is his introduction of the concept of completeness. This is in essence not a restriction of the subject of research, but the isolation of regular realizations of various structural types of separable measures. The result about the inextendability of complete separable measures is very elegant. The limitation to complete spaces implies that the structural isomorphisms of measures are one-to-one pointwise correspondences of the spaces (up to sets of measure zero). Therefore the classification of complete spaces ... coincides with the classification of structural types and has been given exhaustively ... The first part of the dissertation completes the classification of measurable partitions of separable complete measure spaces. ... In the second part, the automorphisms of separable complete measure spaces are investigated. In 1932 von Neumann proved the theorem about the decomposition of automorphisms into transitive ergodic components. However it remained unelucidated to what degree the structure of the components defines the structure of the original automorphism. This problem was also solved by Rokhlin. Besides the solution of this important problem, Rokhlin's exposition has the advantage of methodological purity; in the initial papers measure was considered in topological spaces ... ' We may add that full clarification of the problem of a system of conditional measures (a canonical system of measures) for measurable partitions constitutes the core of Rokhlin's work. Before that, this problem had been discussed repeatedly and not always correctly. Rokhlin showed that in a certain sense it is impossible to speak of this system without completeness; it may not exist. The subsequent development of the theory of dynamical systems and measure theory and, in particular, entropy theory showed how important are the concept of a measurable partition and the corresponding techniques. It is geometric and more convenient than the apparatus of σ-algebras for the study of dynamical systems.

From 1947 to 1951 Rokhlin worked as a junior member of the research staff at the Institute of Mathematics of the Academy of Sciences, in Pontryagin's department. In 1951 he defended a doctor of science dissertation 'On the most important metric classes of dynamical systems'. Bogolyubov, Gel'fand and Kolmogorov served as referees. One of the reviews says, 'The dissertation is an exposition of the whole of metric theory, based on the author's earlier development of measure theory'. The 'Rokhlin lemma', the theorems of mixing, spectral theory, decomposition into ergodic components, etc, are contained in this work. These results found a place in the well-known surveys and articles of the late 1940s. Soon afterwards, recommending Rokhlin to the position of senior member of the research staff or to a professorship at some higher educational institution, influential mathematicians wrote: 'In measure theory and the metric theory of dynamical systems, Rokhlin's results are numbered among the most interesting and significant in the last decade' (Kolmogorov); 'In topology the solution of the urgent and difficult problem of the classification of mappings of an $(n+3)$-dimensional sphere to an n-dimensional one is due to Rokhlin ... his results in this area are the most far-reaching at the present time' (Pontryagin). And both mathematicians later wrote of Rokhlin as 'an outstanding mathematician, making a significant contribution to mathematical science'.

However, despite all this, Rokhlin did not manage to remain at the Institute of Mathematics or to obtain a teaching post at the University or any Higher Education Institute in Moscow! He accepted the offer of a professorship at the Institute of Forestry in the remote city of Arkhangelsk. He worked there until September 1955, and then moved with his family (his wife Anna Aleksandrovna Gurevich-Rokhlin, also a mathematician, had studied at the University of Voronezh and later was a postgraduate student of Pontryagin; her field of study was topological groups and later the theory of dynamical systems; his son Vladimir Rokhlin was born in 1952 and his daughter Liza in 1955) to the town of Ivanovo, not far from Moscow, where he worked as a professor of mathematical analysis at the Pedagogical Institute until September 1957. He then moved even nearer to Moscow to the small town of Kolomna, where he also worked at the Pedagogical Institute until the middle of 1960.

All these years were highly productive for Rokhlin. He was especially active in topology, and returned to ergodic theory in the late 1950s. The reason for this was a remarkable discovery by Kolmogorov: the entropy of dynamical systems. Starting from 1957, Rokhlin led a seminar on ergodic theory at Moscow University (he did not work there), in which many young people took part, who subsequently became well-known mathematicians, including Rokhlin's first graduate student Abramov; the young Moscow mathematicians Sinai and Girsanov, and others. At first the spectral and metric theory of dynamical systems was studied in the seminar, and from 1958 the main topic was entropy.

In 1960 the rector of the University of Leningrad, the well-known geometer A. D. Aleksandrov, concerned about the development of scientific schools at his university (this was not common among rectors at that time) invited Rokhlin to take up the post of professor of geometry at Leningrad; by that time Rokhlin had won prestige as one of the most competent specialists in ergodic theory and geometric and algebraic topology in the Soviet Union, and also fame as an able teacher. Let us note that this was the time of a rapid flowering of topology, but in the Soviet Union there were very few people, who were familiar with the recent achievements in topology in the 1950s. Rokhlin accepted the offer and moved to Leningrad in the autumn of 1960. The Leningrad period, especially at first, was the happiest and most fruitful of Rokhlin's scientific life; unfortunately it ended too soon.

It was during those years that Rokhlin's talent as a mathematician, teacher and administrator, held in check until then by the circumstances of his life, unfolded most notably and earned recognition in the mathematical community of Leningrad – after Moscow, the country's second scientific and mathematical centre.

In 1960 an ergodic seminar, which was to continue in existence for nearly ten years, began to work under his leadership. The way for this seminar had been paved by the interest, which, even before Rokhlin's arrival, some young Leningrad mathematicians showed in measure theory and ergodic theory. In addition, shortly before Rokhlin, his student Abramov, who was one of the first researchers of entropy and later worked in mathematical economics, moved to Leningrad. The membership of this seminar also included Ibragimov, Kagan, Makarov, Sudakov, Vershik and later Yuzvinski, Belinskaya and others. D. Faddeev, Linnik, Sinai, Arnol'd and many others spoke at the seminar. In the late 1960s Rokhlin himself stopped working in ergodic theory. At that time Vershik set up a seminar on smooth systems and the algebraic aspects of measure theory and dynamical systems, which is still in existence and is partly continuing the theme of Rokhlin's seminar. The ergodic seminar worked in close contact with the Moscow seminar of Sinai and V. M. Alexeyev, which continued the traditions of Rokhlin's seminar of the late 1950s. The participants in the seminars often presented reports to one another: Sinai, Katok, Stepin, Gurevich, Oseledets and others spoke repeatedly at Leningrad; and Vershik, Yuzvinski and later younger seminar participants - Livshits, Zakharevich and others - spoke at Moscow.

Rokhlin's topological seminar was even better known. It was set up just afterwards, in 1961, and immediately became one of the country's topological centres. It is necessary to say that Rokhlin immediately started teaching extensive topological courses: in combinatorial, algebraic and smooth topology. These courses attracted huge audiences, among whom were not only undergraduate and postgraduate students, but also professors of the faculty. Thanks to Rokhlin's initiative and persistence, the first compulsory topology course in the Soviet Union was introduced, including the minimum of topological knowledge, which we require of a modern entrant to the Faculty of Mathematics. Rokhlin gave the course until 1974. The topological seminar was an object of Rokhlin's special attention; he was able in a comparatively short space of time to build up a world class topological school, whose products included Gromov, Eliashberg, Viro, Kharlamov and many others. Topologists of the first rank from Moscow and other towns - first of all Novikov, Fuks, Arnol'd - kept close contact with the seminar. At present the seminar continues under the leadership of Viro and Rokhlin's talented students of recent years - Ivanov, Turaev and younger mathematicians - take part in it.

In the early 1970s, Rokhlin led a seminar on Riemannian geometry at the Pedagogical Institute, in which Gromov, Burago and others took an active part. The well-known articles on embeddings of Riemannian manifolds were a result of his work.

For almost fifteen years (until a severe heart attack in 1974) Rokhlin was one of the most active mathematicians in Leningrad. He gave lectures on various subjects, from popular lectures at meetings of the Leningrad and Moscow Mathematical Societies to specialized reviews; was involved in the organization of mathematical education and the problem of school education; led a methodological seminar; etc. He had many postgraduate and undergraduate students; he edited and translated books. He took part in mathematical schools on geometry, topology and dynamical systems; he examined and reviewed the most significant dissertations in topology and ergodic theory. His activity did not slacken much and after his partial recovery (1975-76) he completed what was to become the widely known 'Nachalnyi kurs topologii' (Elementary course of topology) (with Fuks), and planned and partially realized other projects of books and articles. The large role that he played was determined by his outstanding mathematical knowledge and breadth, by his wide understanding of science and his mathematical taste. His advice and recommendations were highly valued by specialists throughout the country.

Other causes besides illness led to deterioration in Rokhlin's position in the University in the second half of the 1970s and especially in the early 1980s. That reflected in an obvious way the general rapid deterioration in the moral climate in the country; the predominance of bureaucracy in science and education, which was hostile to talents and treated what, in its opinion, was of no use to itself with suspicion. Rokhlin's nonconformism was a secret to nobody, though he himself did not take part in any activity other than research and teaching. Doubtless, his 'background' (i.e. the history of captivity; his Jewish origin; the biography of his relatives) played a large role in the administration's dislike of him, both personally and as a scientist. All this ended with his forced retirement in 1980 after reaching the age of 60. Desperate efforts to resist this on the part of some of his friends delayed this decision for only one year; he was permitted to be a consultant professor until 1981. And this at a time when he, though not quite healthy (the effects of the heart attack had not disappeared), was taking an active part in mathematics and teaching and, in particular, had created a new school in real algebraic geometry, well-known at the present time!

In the last years of his life after his retirement Rokhlin continued to work mainly in real algebraic geometry and four-dimensional topology. He planned a series of papers and books on popular and general scientific subjects, discussed problems of education, methodology, publication of literature, etc. Throughout these years, he fought with fortitude against illness; nobody heard a complaint from him, and, perhaps because of this, his death was unexpected and shocked all, who were associated with him; several days before it he was discussing current mathematical problems animatedly with students and colleagues.

What are the reasons for Rokhlin's great influence as a scholar and mathematician? What explains his leading role as a teacher and educator, exerting a great influence on the development of a number of mathematical schools? I think that the answer to this question can be expressed in a few words as follows. First of all Rokhlin was by nature possessed of great and universally recognized mathematical abilities, which always put him on the highest plane in the fields where he worked. His devotion to science, his scientific conscientiousness and moral purity were additional important characteristics, making him at times an arbiter in complicated situations in modern scientific and non-scientific life. But perhaps it is especially important to emphasize his uncompromising attitude in questions of honour and dignity, which is not so often found in our time. But if we speak of values directly related to mathematics, we must here mention the exceptionally good taste and wide understanding of mathematics and science as a whole, that were characteristic of Rokhlin. This feature distinguished Rokhlin at all stages of his mathematical creativity and made him in a certain sense a model of the universal mathematician.

We must not forget his special interest in literature, history and languages; his independent and original judgements always aroused a desire to discuss and to think. Doubtless, his fine style in lectures, reports and articles and especially surveys is a sign of a considerable literary gift. His precise and rich language, his balanced and well thought out judgements, his determination in his actions, the clarity of his values and finally his smartness of appearance - all these are remembered by all who knew him.

Rokhlin's scientific legacy is comparatively small in volume and can be approximately divided into four parts: topology (basically four-dimensional topology and the algebraic apparatus of topology); real algebraic geometry (in his last years); ergodic theory (basically the spectral theory, entropy theory and algebraic aspects); and, lastly, works on history, teaching and the methodology of mathematics. The last group unfortunately did not find sufficient expression in publications, but it continually occupied Rokhlin and his ideas, which he often presented in talks and public lectures, are sufficiently well known and exerted an influence on those around him.

We will here touch only upon the works on ergodic theory. There are 24 publications on this subject; they are listed below; I have added to them a short list of unpublished manuscripts, which were unfortunately not completed and are numbered among Rokhlin's unrealized projects. For example, he repeatedly returned to the idea of writing a book on the metric theory of dynamical systems, including a large section on general measure theory presented in his style; as early as the 1940s he wrote several chapters and proposed to his young co-authors that they continue the work later on, but when he ceased to work on ergodic theory his interest in this idea cooled somewhat. We note that some traces of his intentions were realized in surveys and books by other authors, which were published later on.

Rokhlin introduced into ergodic theory, geometric and algebraic ideas, which, as a result of its earlier, rather analytical origin, it lacked. In this Rokhlin continued the tradition of Von Neumann. In his works the geometric aspect of the matter (partitions, dynamics, etc) predominated over the analytic aspects. He strongly suggested that systems of algebraic, probabilistic, number-theoretic etc, origin be considered simultaneously. This tradition became established and gave excellent results in the works of his students and followers. On the other hand he strictly followed the axiomatic constructions of measure theory, in which the influence of Kolmogorov and Von Neumann was also expressed.

Lebesgue spaces, introduced by Rokhlin, in his student paper and studied in detail in subsequent dissertations and in the article, 'On the fundamental ideas of measure theory', turned out to be a very successful concept, and Rokhlin's proposed axiomatics turned out to be an exceptionally convenient refinement of the previous axiomatizations. It is fair to say that without doubt after the axiomatics of Kolmogorov and Von Neumann, Rokhlin made the most important step toward isolating a proper category of measure spaces. Unfortunately the convenience and importance of Lebesgue spaces was far from being immediately realized. Perhaps researchers for a long time used topological concepts for constructions, which were in essence purely metric. At the present time there is no doubt that the category of Lebesgue spaces is a basic structure of ergodic theory, measure theory, etc. It should be noted in passing that the important observation that a system of conditional measures, or, as he put it, a canonical system of measures, exists only for measurable partitions of Lebesgue spaces and that attempts to introduce it into other categories are incorrect is also due to Rokhlin.

The Rokhlin lemma, proved independently by Halmos in a somewhat different variant, has been written about in detail in the article by Weiss. This lemma and its generalizations constitute the basis of the contemporary theory of approximations.

Let us also note that in an early paper on decompositions of an automorphism into its ergodic components Rokhlin in effect proved a variant of the theorem about measurable choice, which is now called the Rokhlin-Kuratovski-Ryl'-Nardzevski theorem. Under the influence of the Gel'fand-Neumark-Raikov-Shilov theory of normed rings, Rokhlin undertook a not fully completed attempt to construct a theory of the so-called unitary rings. This theory is dual to the theory of Lebesgue spaces, representing its functional-analytic variant. It turned out to be convenient for some problems; for example in my papers on the theory of Gaussian systems, undertaken on Rokhlin's initiative in the early 1960s, it helped to give an axiomatic construction of such systems. I will also note that it was on his initiative that I undertook the study of measurable flows, continuing his work on flows with a discrete spectrum. Its result was the proof of the existence of a measurable realisation for continuous flows and even any continuous actions of locally compact groups. Rokhlin's works on mixing are generally known; it is curious that he already conceived the idea of multiple mixings before the war and he had hoped to construct examples, distinguishing mixing of different orders. It seems that soon such examples will finally be found. The problems and questions that Rokhlin posed from time to time were especially fruitful for researchers. One of these problems was the problem of a trajectory isomorphism. Rokhlin initially suggested that a trajectory isomorphism preserves some invariants and, in particular, entropy. The problem was posed for the first time in the late 1950s for various irrational rotations of a circle. At that time Dye's papers were not known or were not linked to this theme, inasmuch as they were written in quite different traditions. After Belinskaya constructed a dyadic approximation to an arbitrary automorphism with the aid of skew products in 1966 (it can also be constructed more simply with the aid of the refined Rokhlin-Halmos lemma, which was done by me later on), I began to work on the problem of dyadic sequences, which led to the proof of the isomorphism of tame partitions and the trajectory isomorphism of any ergodic automorphism. This proof differs substantially from Dye's proof, if the latter is translated into compatible terms and it was obtained independently, although somewhat later. The later history of trajectory theory has been published more than once, but perhaps not always with sufficient completeness. Rokhlin followed the development of the trajectory theory of nonmeasurable partitions with interest, as he considered it to be the natural continuation of the theory of measurable partitions.

The papers of the entropy period are discussed in detail in Yuzvinski's paper and we shall not touch upon them here. We must, however, emphasise one circumstance. Kolmogorov used the ideas and language of the theory of measurable partitions in his paper on entropy. I think that an accurate analysis of the concept of the entropy of dynamical systems, developed in the Kolmogorov–Sinai cycle of papers and then by Rokhlin, Abramov, Pinsker, etc. is simply impossible without the theory of measurable partitions, and in particular without that section of it which relates to decreasing sequences. A small and insignificant mistake in Kolmogorov's first paper, discovered by Rokhlin, which led to the necessity of giving another definition of entropy, according to Sinai, is linked to some intricacies of this theory (see the note in Kolmogorov's second paper). The unity of these two definitions was restored after Rokhlin, significantly later on, proved the theorem about generators for aperiodic automorphisms. The final results here belong to Krieger. There had been attempts (by Arov) to construct an entropy theory, which were not realized, due to the author's lack of the techniques of measurable partitions.

In his later years, during the period of completing his work on ergodic theory, Rokhlin returned to problems of the theory of invariant partitions for automorphisms; we will describe these unpublished works. Their beginning was the classic paper with Sinai, in which in particular they proved the coincidence of K-automorphisms and automorphisms with completely positive entropy (in one direction this had been proved earlier by Pinsker). The first formulae of entropy theory – the Abramov formula for an induced automorphism; the formulae for automorphisms of compact groups (Sinai, Arov, Yuzvinski, etc) – were a development of Rokhlin's ideas and conjectures. This 'golden age' of development (1958–1966) occurred to a significant degree under Rokhlin's influence. His interest in ergodic theory gradually abated after the appearance of post-entropy ideas: approximations and especially the stream of papers by Ornstein and his successors. Rokhlin was doubtless interested in the course of events, but he did not take part in them, especially as they were outweighed by his topological interests.

I will now make note of two circumstances. Rokhlin, over a long period of time, had been interested in number theory and in the possibility of applying ergodic theory to it. Although there is only one paper by him on this subject and that is simultaneously devoted to the theory of exact endomorphisms, namely the paper on continued fractions and the Gauss endomorphism, he considered (and this opinion found indirect confirmation in, for example, Linnik's paper on the ergodic method in number theory), that the possibilities of metric theory in number theory had not yet been exhausted.

The other circumstance is connected with smooth and classical dynamics. It might appear strange that he, an outstanding specialist in the field of smooth manifolds, who knew classical physics and dynamics well, did not attempt to connect ergodic theory with smooth dynamics, especially as many of his students and those who were close to him or experienced his influence took an active part in it (Sinai, Arnol'd, Anosov, etc). Moreover the seminar repeatedly included communications about the work of Smale, Anosov, etc. Rokhlin himself said that he preferred 'cleanly' formulated problems, in which completely dissimilar categories were not mixed. In other words, he considered smooth and metric dynamics to be nonmixable domains. Possibly, this point of view manifested echoes of an axiomatic rigorism, which is doubtless not popular nowadays, but it is impossible to deny its consistency.

I have not aimed in this outline to analyse Rokhlin's creative work in the field of ergodic theory from all sides; doubtless people will still return to its analysis. Two interesting surveys by Weiss and Yuzvinski follow this article, Weiss's work is particularly interesting in that it expresses the view of a highly qualified specialist on Rokhlin's work from 'outside' his school. It is very characteristic and it concurs with our ideas that specialists in ergodic theory in the West, as the writer says, have for a long time had a tendency to use only finite partitions and have not fully appreciated the depth of the theory of measurable partitions as conceived by Kolmogorov and created by Rokhlin. However, as I have already written, obviously neither entropy theory nor other sections of ergodic theory in their deepest parts can be expounded without this theory. On the other hand a synthesis of the theory of measurable partitions in all its generalities, and combinatorial techniques has been so brilliantly presented in the recent works of the Ornstein–Weiss school and researchers on the theory of approximations have returned us to the beginnings of probability theory and measure theory, converting contemporary ergodic theory into 'limiting combinatorics'.

Among the sources of ergodic theory as a mathematical discipline stand the names of Von Neumann and Kolmogorov; after them we can name only a few, who from the 1930s to the 1950s developed this theory and gave it a contemporary form: these are Birkhoff, Khinchin, Hopf, Kakutani, Halmos and Vladimir Abramovich Rokhlin.

ROKHLIN'S WORKS ON MEASURE THEORY AND ERGODIC THEORY

[1] On a classification of measurable partitions. *Doklady Akad. Nauk SSSR* **58** (1947), 29–32.

[2] On the problem of classification of the automorphisms of Lebesgue spaces. *Doklady Akad. Nauk SSSR* **58** (1947), 189–191.

[3] Unitary rings. *Doklady Akad. Nauk* **59** (1948), 643–646.

[4] A 'general' measure-preserving transformation is not mixing. *Doklady Akad. Nauk* **60** (1948), 349–351.

[5] On the fundamental ideas of measure theory. *Mat. Sbornik* **25** (67) (1949), 107–150.
= On the fundamental ideas of measure theory. *Amer. Math. Soc. Translation* **71** (1952) (1) 10, 1–54.

[6] On the decomposition of a dynamical system into transitive components. *Mat. Sbornik* **25** (67) (1949), 235–249.

[7] On dynamical systems whose irreducible components have a pure point spectrum. *Doklady Akad. Nauk SSSR* **64** (1949), 167–169.

[8] On the approximation of non-periodic flows by periodic ones. *Doklady Akad. Nauk SSSR* **64** (1949), 619–620. (With A. A. Gurevich.)

[9] On endomorphisms of compact commutative groups. *Izvestiya Akad. Nauk SSSR Ser. Mat.* **13** (1949), 329–340.

[10] Selected topics from the metric theory of dynamical systems. *Uspehi Matem. Nauk* **4** No. 2 (30) (1949), 57–128.

[11] Approximation theorems for measurable flows. *Izvestiya Akad. Nauk SSSR Ser. Mat.* **14** (1950), 537–548. (With A. A. Gurevich.)

[12] Metric classification of measurable functions. *Uspehi Matem. Nauk* **12**, No. 2 (74) (1957), 169–174.

[13] Spectral theory of dynamical systems. *Trudy Tret'evo Vsesoyuznovo Matematicheskovo S'ezda 1956* (Proc. Third All-Union Congress 1956), Vol. 3, 284–292. (With S. V. Fomin.)

[14] Entropy of metric automorphism. *Doklady Akad. Nauk SSSR* **124** (1959), 980–983.

[15] New progress in the theory of transformations with invariant measure. *Uspehi Matem. Nauk* **15** No. 4 (94) (1960), 3–26.
= New progress in the theory of transformations with invariant measure. *Russian Math. Surveys* **15**, no. 4, 1–22.
[16] Construction and properties of invariant measurable partitions. *Doklady Akad. Nauk SSSR* **141** (1961), 1038–1041. (With Ya. G. Sinai.)
[17] The entropy of an automorphism of a compact commutative group. *Teor. Veroyatnost. i Primenen.* **6** (1961), 351–352.
[18] Exact endomorphisms of a Lebesgue space. *Izv. Akad. Nauk SSSR Ser. Mat.* **25** (1961), 499–530.
[19] Entropy of a skew product of mappings with invariant measure. *Vestnik. Leningrad. Univ.* **17** (7) (1962), 5–13. (With L. M. Abramov.)
[20] An axiomatic definition of the entropy of a transformation with invariant measure. *Doklady Akad. Nauk SSSR* **148** (1963), 779–781.
[21] Generators in ergodic theory. *Vestnik. Leningrad. Univ.* **18** (1) (1963), 26–32.
[22] Metric properties of endomorphisms of compact commutative groups. *Izv. Akad. Nauk SSSR* **28** (1964), 867–874.
[23] Generators in ergodic theory II. *Vestnik. Leningrad. Univ.* **20** (13) (1965), 68–72.
[24] Lectures on the entropy theory of transformations with invariant measure. *Uspehi Matem. Nauk* **22** No. 5 (137) (1967), 3–56.

LIST OF V. A. ROKHLIN'S MANUSCRIPTS ON ERGODIC THEORY

[1] A notebook of small format with black binding without a title, about 80 pages; material for a book (apparently 1940s.) Table of Contents:

Part I. Lebesgue spaces.

Chapter 1. Basic definitions.
Chapter 2. Measurable partitions.
Chapter 3. The normed structure associated with a Lebesgue space.
Chapter 4. The unitary ring associated with a Lebesgue space.
Chapter 5. Compact representations of Lebesgue spaces.
Chapter 6. Spaces of infinite measure.
Chapter 7. Applications to the theory of characters of continuous groups.

Part II. Lebesgue spaces in motion.

Chapter 8. Canonical decomposition. Spectral methods.
Chapter 9. General theorems about ergodic automorphisms. (Approximation methods.)
Chapter 10. Topological methods.
Chapter 11. Some special classes of automorphisms.
Chapter 12. Groups of automorphisms.
Chapter 13. Automorphisms of spaces with continuous measure.

Also a plan of the first chapter by paragraphs and drafts of various parts. The text occupies the whole notebook.

[2] A draft: 'Preface'. Organisation of the book. History of the theory of transformations with invariant measure, its connections and applications. Measure theory as an independent science and the proper place of the theory of transformations with invariant measure. What is usually understood as measure theory; the purpose of Chapter 1; what is considered to be known. The main aim of the book new entities. With what to begin. The characteristics of old parts. The choice of material. The level of generality, Lebesgue space.
[3] A manuscript: 'Transformations with invariant measure." Part of a 'book'. 59 pages +29 probably added later. It probably dates from the 1960s.
[4] A manuscript: 'Unitary rings'. 18 pages, probably from the 1950s.
[5] Ergodic theory 1966–1967. Lectures (plans). 4 pages.
[6] Invariant partitions, June 1967. 2 pages added in July and September 1967.
[7] 'Closed partitions. Report 1.10.1968'. 1 page added 11.10 and 21.10.68.
[8] 'Saturated partitions. December 1969.' 2 pages. 24 items.

(*Translated by Ann Dowker and Alex Sosinsky, June 1989*)