Icy Bodies of the Solar System
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Simulations for Terrestrial Planets Formation

Jianghui Ji[1] **and Niu Zhang**[1,2],

[1]Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
email: jijh@pmo.ac.cn

[2]Graduate School of Chinese Academy of Science, Beijing 100049

Abstract. We investigate the formation of terrestrial planets in the late stage of planetary formation using two-planet model. At that time, the protostar has formed for about 3 Myr and the gas disk has dissipated. In the model, the perturbations from Jupiter and Saturn are considered. We also consider variations of the mass of outer planet, and the initial eccentricities and inclinations of embryos and planetesimals. Our results show that, terrestrial planets are formed in 50 Myr, and the accretion rate is about 60% - 80%. In each simulation, 3 - 4 terrestrial planets are formed inside "Jupiter" with masses of $0.15 - 3.6 M_{\oplus}$. In the 0.5 - 4AU, when the eccentricities of planetesimals are excited, planetesimals are able to accrete material from wide radial direction. The plenty of water material of the terrestrial planet in the Habitable Zone may be transferred from the farther places by this mechanism. Accretion may also happen a few times between two giant planets only if the outer planet has a moderate mass and the small terrestrial planet could survive at some resonances over time scale of 10^8 yr.

Keywords. methods:n-body simulations-planetary systems-planetary formation

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1. Introduction

The discovery of the extrasolar planets (Mayor & Queloz (1995), Lee & Peale (2002), Ji *et al.* (2003)) around solar-type stars indeed provides substantial clues for the formation and origin of our own solar system. According to standard theory Safronov (1969), Wetherill (1990), Lissauer (1993), it is generally believed that planet formation may experience such several stages: in the early stage, the dust grains condense to grow km-sized planetesimals; in the middle stage, Moon-to-Mars sized embryos are created by accretion of planetesimals. When the embryos grow up to a core of $\sim 10 M_{\oplus}$, runaway accretion may take place. With more gases accreted onto the solid core, the embryos become more massive and eventually collapse to produce giant Jovian planets (Ida & Lin (2004)). At the end of the stage, it is around that the protostar has formed for about 3 Myr, the gas disk has dissipated. A few larger bodies with low e and i are in crowds of planetesimals with certain eccentricities and inclinations. In the late stage, the terrestrial embryos are excited to high eccentricity orbits by mutual gravitational perturbation. Next, the orbital crossings make planets obtain material in wider radial area. In this sense, solid residue is either scattered out of the planetary system or accreted by the massive planet, even being captured (Nagasawa & Ida (2000)) at the resonance position of the giant planets.

Chambers (2001) made a study of terrestrial planet formation in the late stage by numerical simulations, who set $150 - 160$ Moon-to-Mars size planetary embryos in the area of $0.3 - 2.0$ AU under mutual interactions from Jupiter and Saturn. He also examined two initial mass distributions: approximately uniform masses, and a bimodal mass distribution. The results show that $2 - 4$ planets are formed within 50 Myr, and finally survive over 200 Myr timescale, and the final planets usually have eccentric orbits with higher eccentricities and inclinations . Raymond, Quinn & Lunine (2004), Raymond, Quinn & Lunine (2006) also investigated the formation of terrestrial planets. In the simulations, they simply took into account Jupiter's gravitational perturbation, and the distribution of material are in $0.5-4.5$ AU. Their results confirm a leading hypothesis for the origin of Earth's water: they may come from the material in the outer area by impacts in the late stage of planet formation. Raymond, Mandell, & Sigurdsson (2006) explored the planet formation under planetary migration of the giant. In the simulations, super Hot Earth form interior to the migrating giant planet, and water-rich, Earth-size terrestrial planet are present in the Habitable Zone ($0.8-1.5$ AU) and can survive over 10^8 yr timescale.

In our work, we consider two-planet model, in which Jupiter and Saturn are supposed to be already formed, with two swarms of planetesimals distributed in the region among $0.5-4.2$ AU and $6.2-9.6$ AU respectively. The initial eccentricities and inclinations of planetesimals are considered. We also vary the mass of Saturn to examine how the small bodies evolve. The simulations are performed on longer timescale 400 Myr in order to check the stability and the dynamical structure evolution of the system. In the following, we briefly summarize our numerical setup and results.

2. Numerical Setup

The timescale for formation of Jupiter-like planets is usually considered to be less than 10 Myr Briceño *et al.* (2001), the formation scenario of planet embryos is related to their heliocentric distances and the initial mass of the star nebular. If we adopt the model of 1.5 MMSN (Minimum Mass Solar Nebular), the upper bound of the timescale for Jupiter-like planet formation corresponds to the timescale for embryo formation at 2.5 AU Kokubo & Ida (2002), which is just at $3:1$ resonance location of Jupiter. In the region $2.5-4.2$ AU, embryos will be cleared off by strong perturbation from Jupiter. There should be some much smaller solid residue among Jupiter and Saturn, even though the clearing effect may throw out most of the material in this area. We set embryos simply in the region $0.5-2.5$ AU and planetesimals at $0.5-4.2$ AU and $6.2-9.6$ AU.

We adopt the surface density profile as follows (Raymond, Quinn & Lunine (2004)):

$$\Sigma(r) = \begin{cases} \Sigma_1 r^{-3/2}, & r < snow\ line, \\ \Sigma_{snow}(\frac{r}{5AU})^{-3/2}, & r > snow\ line. \end{cases} \tag{2.1}$$

In (2.1), $\Sigma_{snow} = 4\ g/cm^2$ is the surface density at snowline, where the snowline is at 2.5 AU with $\Sigma_1 = 10\ g/cm^2$. The mass of planetary embryos is proportional to the width of the feeding zone, which is associated with Hill Radius, R_H, so the mass of an embryo increases as

$$M_{embryo} \propto r\Sigma(r)R_H \tag{2.2}$$

The embryos in the $0.5-2.5$ AU are spaced by Λ (Λ varying randomly between 2 and 5) mutual Hill Radii, $R_{H,m}$, which is defined as

$$R_{H,m} = (\frac{a_1+a_2}{2})(\frac{m_1+m_2}{3M_\odot})^{1/3} \tag{2.3}$$

where $a_{1,2}$ and $m_{1,2}$ are the semi-major axes and masses of the embryos respectively. Replacing R_H in (2.2) with $R_{H,m}$, and substituting (2.1) in (2.2), then, we achieve a

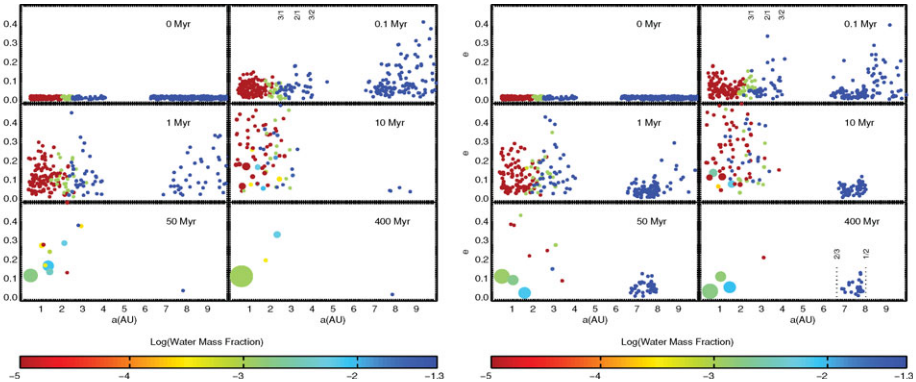

Figure 1. *Left panel*: (a) Snapshot of simulation 2a with $M_{Saturn} = 5M_{\oplus}$. The total mass of embryos is $2.4M_{\oplus}$, the masses of planetesimals inside Jupiter are $0.0317M_{\oplus}$, and those outside Jupiter are $0.0375M_{\oplus}$. Planetesimals among Jupiter and Saturn were nonself-gravitational. Note the size of each object is relative, and the value bar is log of water mass fraction. *Right panel*: (b) snapshot for similuation 2b.

relation law between the mass of embryos and the parameter Λ as

$$M_{embryo} \propto r\Sigma(r)R_{H,m} \propto r^{3/4}\Lambda^{3/2}\Sigma^{3/2} \tag{2.4}$$

Here, we equally set the masses of planetesimals inside and outside Jupiter, respectively. Consequently, the number distribution of the planetesimals is simply required to meet $N \propto r^{-1/2}$. Additionally, we remain the total number of planetesimals and embryos inside Jupiter, and the number of planetesimals outside Jupiter both equal to 200. The mass inside and outside Jupiter is equal to be $7.5M_{\oplus}$. The eccentricities and inclinations vary in $(0 - 0.02)$ and $(0 - 0.05°)$, respectively. The mass of Saturn in simulations 1a/1b, 2a/2b and 3a/3b are $0.5M_{\oplus}$, $5M_{\oplus}$, $50M_{\oplus}$ respectively. Each simulations marked by label a (or b) is run to consider (not consider) self-gravitation of planetesimals among giants.

We use the hybrid symplectic integrator (Chambers (1999)) in MERCURY package to integrate all the simulations. In addition, we adopt 6 days as the length of time step, which is a twentieth period of the innermost body at 0.5 AU. All runs are carried out over 400 Myr time scale. At the end of the intergration, the changes of energy and angular momenta are 10^{-3} and 10^{-11} respectively. Six simulations are performed on a workstation composed of 12 CPUs with 1.2 GHz, and each costs roughly 45 days.

3. Results

Fig. 1(a) is a snapshot of simulation 2a. At 0.1 Myr, it is clear that the planetesimals are excited at the 3 : 2 (3.97 AU),2 : 1 (3.28 AU) and 3 : 1 (2.5 AU) resonance positions with Jupiter, and this is quite similar to the Kirkwood gaps of the main asteroidal belt in solar system. For about 1 Myr, planetesimals and embryos are deeply intermixed, where most of the bodies have stirred to be large eccentricities. Collisions and accretions frequently emerge among planetesimals and embryos. This process continues until about 50 Myr, and the planetary embryos are mostly generated. The formation timescale of embryos is in accordance with that of Ida & Lin (2004). Finally, inside Jupiter, 3 terrestrial planets are formed with masses of $0.15 - 3.6M_{\oplus}$. However, at the outer region, the planetesimals are continuously scattered out of the system at 0.1 Myr. For about 10 Myr, there are no survivals except at some resonances with the giant planet. As shown in the Figure, there is a small body at the 1 : 2 resonance with Jupiter. Due to scattering amongst

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Table 1. Properties of terrestrial planets from different systems

System	accretion rate	n	$\bar{m}(m_\oplus)$	concentration	$\bar{e}$	$\bar{i}(^\circ)$
1a	73.2518%	3	1.8313	0.4606	0.1381	7.6963
1b	80.3853%	3	2.0096	0.4262	0.0937	1.7790
2a	59.8322%	3	1.4958	0.8116	0.2108	16.9117
2b	72.9779%	4	1.3683	0.4299	0.0999	5.1415
3a	65.1098%	3	1.6277	0.5337	0.2063	5.9153
3b	66.9694%	3	1.6742	0.5040	0.1839	5.2447
1a-3b	69.7544%	3.2	1.6678	0.5276	0.1554	7.1148
solar	-	4	0.4943	0.5058	0.0764	3.0624

planetesimals, Jupiter (Saturn) migrates inward (outward) 0.13 AU (1.19 AU) toward the sun respectively. Such kind of migration agrees with the work of Fernandez & Ip (1984). Hence, the 2 : 5 mean motion resonance is destroyed, then the ratios of periods between Jupiter and Saturn degenerate to 1 : 3. Therefore, the ratio of periods for Jupiter, small body and Saturn is approximate to 1 : 2 : 3. In the $0.5 - 4$ AU, when the eccentricities of planetesimals are excited, planetesimals are able to accrete material from wide radial direction. The plenty of water material of the terrestrial planet in the Habitable Zone may be transferred from the farther places by this mechanism.

Fig. 1(b) is illustrated for simulation 2b. In comparison with Fig. 1(a), it is apparent that planetesimals are excited more quickly at the 3 : 2 (3.97 AU), 2 : 1 (3.28 AU) and 3 : 1 (2.5 AU) resonance location with Jupiter. The several characteristic timescales are the same as simulation 2a for the bodies within Jupiter. 4 planets are formed in simulation 2b, the changes of position of Jupiter and Saturn behaves like simulation 2a. We point out that simulations 2a and 2b share the initial conditions, but the only difference in them is whether we consider the self-gravitation among the outer planetesimals. There is a little gathered planetesimals survival over 400 Myr among $7 - 8$ AU, located in the area of 2 : 3 (6.63 AU) and 1 : 2 (8.03 AU) resonances with Jupiter. The detailed results for whole simulations that the reader may refer to Zhang & Ji (2009).

The production efficiency of the terrestrial planet in our model is high, and the accretion rate inside Jupiter is $60\% - 80\%$ in the simulations. $3 - 4$ terrestrial planets formed in 50 Myr. 5 of 6 simulations have a terrestrial planet in the Habitable Zone ($0.8 - 1.5$ AU). The planetary systems are formed to have nearly circular orbit and coplanarity, similar to the solar system (see Table 1). We suppose that the above characteristics are correlated with the initial small eccentricities and inclinations. The concentration in Table 1 means the ratio of maximum terrestrial planet formed in the simulation and the total terrestrial planets mass. It represents different capability on accretion. The average value of this parameter is similar to the solar system. Considering the self-gravitation of planetesimals among Jupiter and Saturn, the system has a better viscosity, so that the planetesimals will be excited slower. The consideration of self-gravitation may not change the formation time scale of terrestrial planets, but will affect the initial accretion speed and the eventual accretion rate.

4. Summary and Discussion

We simulate terrestrial planet formation by using two-planet model. In the simulations, the variations of the mass of outer planet, the initial eccentricities and inclinations of embryos and planetesimals are also considered. The results show that, during the terrestrial planet formation, planets can accrete material from various areas inside Jupiter.

Among 0.5 − 4.2 AU, the accretion rate of terrestrial planet is 60% − 80%, i.e., about 20% − 40% initial mass is removed during the progress. The planetesimals will improve efficiency of accretion rate for certain initial eccentricities and inclinations, and this also makes the newly-born terrestrial planets have lower orbital eccentricities. It also indicates that in the planet formation that water-rich terrestrial planet may be formed in the Habitable Zone. Most of the planetesimals among Jupiter and Saturn are scattered out of the system, and such migration induced by scattering (Fernandez & Ip (1984)) or long-term orbital evolution can make smaller bodies capture at some mean motion resonance location. Accretion could also happen a few times between two planets if the outer planet owns a moderate mass, and a small terrestrial planet could survive at some resonances over time scale of 10^8 yr. The outcomes further reveal that the outer planet has little effect on dynamical architecture inside Jupiter.

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