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GEOMETRY OF INTEGRABLE LATTICE EQUATIONS AND THEIR REDUCTIONS

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Modern research into discrete integrable systems has provided new insights into a wide variety of fields, including generalisations of special functions, orthogonal polynomials and dynamical systems theory. In this thesis, we extend one of the most productive insights in this area to higher dimensions. In particular, we show how to apply ideas from resolution of singularities and birational geometry to discrete systems in higher dimensions.

The most widely studied setting for these ideas lies in spaces of dimension two. By blowing up at certain points to resolve singularities found in maps on surfaces, new surfaces are constructed on which the map becomes an isomorphism, a so-called *space of initial conditions*. This has led to new developments in the field, including the discovery of new examples of integrable maps by Sakai [3] with solutions that have unexpectedly rich properties.

On the other hand, this geometric approach has never been applied to integrable partial difference equations (often called lattice equations), which share other properties with the maps in dimension two. In this thesis, we overcome this gap.

In particular, we examine spaces of initial conditions for integrable lattice equations, which are members of the equations classified by Adler *et al.* [1], known as ABS equations. By explicitly calculating the induced map on their resolved initial value spaces, we find transformations to new lattice equations, and hence find novel reductions to discrete Painlevé equations. We also show that an equation arising from the geometry of ABS equations is satisfied by the coefficients of a cluster algebra associated with a form of the discrete modified Korteweg–de Vries equation. This work can be found in [2].

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