

The volume concludes with a valuable historical essay, whose fluent prose affords a welcome contrast to the austere mathematical style of the main part of the book.

W. LEDERMANN

SALZER, H. E., RICHARDS, C. H. AND ARSHAM, I., *Table for the solution of Cubic Equations* (McGraw-Hill Book Company, New York, 1958), 161 pp., 50s.

The three roots of the cubic equation

$$ax^3 + bx + c = 0$$

can be written in the form

$$-(c/b)f_1(\theta), \quad -(c/b)f_2(\theta), \quad -(c/b)f_3(\theta),$$

where  $f_1(\theta)$ ,  $f_2(\theta)$ ,  $f_3(\theta)$  are simple functions of  $\theta = ac^2/b^3$ . The present table gives  $f_1(\theta)$ ,  $f_2(\theta)$ ,  $f_3(\theta)$  for  $1/\theta = -0.001$  ( $-0.001$ ) $-1$ , for  $\theta = -1(0.001)1$ ;  $1/\theta = 1(-0.001)$   $\cdot 001$  to seven decimals. Each of the three tabulated functions is provided with a table of first and second differences.

The tables should be particularly useful to the occasional computer having only a desk-calculator at his disposal.

I. N. SNEDDON

HILTON, P. J. AND WYLIE, S., *Homology Theory, an introduction to Algebraic Topology* (Cambridge University Press, 1960), 484 pp., 75s.

Only a slight familiarity with analytic topology and group theory is assumed in this book; the prerequisites are summarised at the beginning. As well as giving (i) the basic properties of the simplicial and singular homology groups and cohomology rings, the book introduces (ii) the fundamental group, (iii) covering spaces, (iv) obstruction theory, (v) homological algebra, (vi) spectral sequences. (The elementary properties of homotopy groups, used in (iv) and elsewhere, are summarised.) The exposition is leisurely and is enriched by many discussions of related topics; and there are many exercises, some easy, some not. We mention a few of the related topics to which the main theories are applied: lens spaces are classified by homotopy type as well as by topological type; the cap product is introduced and its connection with intersections indicated; the Hopf invariant is discussed by means of the cohomology ring; spectral homology sequences are applied to homological algebra and to homotopy theory.

The book is intended for the beginner as well as the more advanced student, so I am sure it is right to put simplicial homology theory before any other sort of homology theory. However in other respects the arrangement of the contents seems unnecessarily difficult for the beginner and I suggest a rearrangement leading more easily to the heart of the subject. Although the book aims to "be modern", 1.2-1.5 and 1.7-1.9 seem so old fashioned that I advocate reading pp. 56-65 of [14] (see bibliography) instead; let the beginner have as neat a presentation of simplicial complexes and maps as possible. (Incidentally, simplicial approximation is more perspicuous when the  $V$ 's of p. 337, or better of [12] 1948-9, are used.) Now liberalise the definition of pseudo-simplicial complex in 1.11 by allowing identification of closed simplexes under linear homeomorphism; a pseudo-dissection of a torus is then possible using only two triangles. (A pseudo-simplex may have some of its proper faces identified. Two barycentric subdivisions may be needed to obtain a simplicial from a pseudo-simplicial complex. A pseudo-simplicial complex is best defined as a

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pair  $(K, f)$  where  $K$  is a simplicial complex and  $f$  is an identification map of  $|K|$  onto a space  $X$  which maps each open simplex of  $K$  homeomorphically and is such that if  $f(|s_p^i|)$  and  $f(|s_q^j|)$  meet, then  $p = q$  and  $f$  identifies  $|\bar{s}_p^i|$  and  $|\bar{s}_q^j|$  under linear homeomorphism; the chain groups of  $(K, f)$  are obvious quotients of those of  $K$ . One can adapt the simplicial approximation theorem to pseudo-simplicial complexes; then the latter will have the advantage over block dissections (p. 128) that they can be used for cup products and the fundamental group, and blocks might be omitted on a first reading.

Sections 2.1-2.3 are disappointing, but the beginner must learn the notation of the book for chains, etc. Cohomology is introduced in an admirable way (2.6, 2.7); it is called *contrahomology* in the book because it transforms contravariantly (with respect to maps of spaces). The treatment of relative homology and exact sequences (2.8, 2.9) is excellent. Having read to p. 116, one can justify pseudo-dissection by acyclic carriers (as suggested on p. 133); block-dissections and their application to real projective space pp. 128-135) could be omitted. Cup products are introduced on pp. 140-152 with interesting applications to maps of spheres.

I believe the beginner should next turn to p. 313 and study singular homology. On first reading, normalised and cubical theories might be omitted. After 8.1 (for which a little of 5.6 is needed) one can read pp. 330-40 and 361-4.

Pages 228-246 deal with the fundamental group and lead naturally to pp. 345-49 (for which pp. 318-329 are needed) where the Hurewicz isomorphism theorem is proved. Having got so far the beginner will be able to find his own way about the book.

There are several departures from customary notation. As already mentioned cohomology is called *contrahomology* in this book; it seems hardly worth making the change, for who will forget that cohomology is contravariant? The cohomology ring is denoted by  $R^*(X)$  instead of  $H^*(X)$ ; the same letter  $J$  is used for the additive group of integers and for the ring of integers, so why not similarly in cohomology? It is also confusing to find  $V^n$  used for the closed unit  $n$ -cell. Maps of spaces are written on the right, so  $Xfg$  is the image of  $X$  by the map  $f$  followed by  $g$ .

The printing is very good.

The book achieves the purpose of providing an introduction which reaches the developing parts of the subject, and for those who already know a little algebraic topology is by far the best textbook for further study.

D. G. PALMER

KREYSZIG, E., *Differential Geometry* (Mathematical Expositions No. 11, Toronto, and Oxford University Press, 1959), pp. xiv + 352, 48s.

Differential Geometry is today one of the most popular subjects of research. It experienced a decline after the impetus from General Relativity in the 1920's and 30's had died down, but it has now received new life, mainly through the union with Topology from which there emerged the two closely related subjects of Differential Topology (the study of differentiable manifolds) and Differential Geometry in the Large. One change is that the various classical techniques, based largely on tensor calculus, which were at one time an indispensable part of differential geometry, are now out of fashion and have been replaced by new "suffix-free" methods. Nevertheless the older tools still have their uses, and anyone who wishes to research in geometry or its applications is well advised to have some knowledge of tensor calculus and of the classical differential geometry with which it is so closely associated. It often happens that a new result is first discovered through old-style calculations which are afterwards replaced by an argument along more modern lines.

Because of the popularity of modern differential geometry many universities are