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ERRATUM

DIFFERENTIAL MODULES ON p-ADIC POLYANNULI—ERRATUM

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The statement of Theorem 2.7.6 in the indicated paper is incorrect. For instance, if $m = 0$ (i.e., the base field K contains no additional derivations), it is inconsistent with Theorem 2.7.4 due to the distinction between intrinsic and extrinsic generic radii of convergence. Theorems 2.7.12 and 2.7.13 are incorrect for similar reasons.

We will show here that all of these results become true under the following additional hypothesis.

Hypothesis 1. Recall that the base field K has been assumed (in [**2**, Hypothesis 2.0.1]) to be a complete nonarchimedean field with residue field k of characteristic 0, equipped with derivations $\partial_1, \dots, \partial_m$ which are of rational type with respect to the parameters $u_1, \dots, u_m \in K$. We assume further that for each $z \in K^\times$ with $|z| \neq 1$, there exists an index $j \in \{1, \dots, m\}$ with $|u_j \partial_j(z)| = |z|$.

Remark 2. The last condition in Hypothesis 1 is satisfied, for example, by the complete discretely valued field $K = k((T))$ with $\partial_1 = \mathrm{d}/\mathrm{d}T$.

We must check some stability properties of this hypothesis.

Lemma 3. *Hypothesis 1 is preserved by replacing K by a finite extension K'.*

Proof. To check the condition of the hypothesis for some $z \in (K')^\times$ with $|z| \neq 1$, we may check it instead for z^h for some positive integer h. We may thus assume that there exists $x \in K$ with $|x| = |z|$. Let $P(T) \in \mathfrak{o}_K[T]$ be a monic polynomial lifting the minimal polynomial over k of the image of z/x in k'. Put $Q(T) = P(T/x) = Q_d T^d + Q_{d-1} T^{d-1} + \cdots + Q_0$, so that $|Q(z)| < 1$ and $|zQ'(z)| = 1$.

Choose an index i for which $|Q_i z^i| = 1$, then choose j for which $|u_j \partial_j(Q_i)| = |Q_i|$. From the definition of P, we deduce that $|\sum_{h=0}^{d-1} u_j \partial_j(Q_h) z^h| = 1$; however, since $Q(z)$

maps to zero in k, so does

$$u_j \partial_j (Q(z)) = u_j \partial_j (z) Q'(z) + \sum_{h=0}^{d-1} u_j \partial_j (Q_h) z^h.$$

Since $|zQ'(z)| = 1$, we deduce that $|u_j \partial_j (z)| = |z|$, as desired. □

Lemma 4. *For any $\rho > 0$, Hypothesis 1 is preserved by replacing K by the completion F_ρ of $K(t)$ for the ρ-Gauss norm.*

Proof. To check the hypothesis for $z \in F_\rho^\times$ with $|z| \neq 1$, first choose $x \in K$ with $|z| = |x|$. We can then write z/x as a formal sum $\sum_{i \in \mathbb{Z}} c_i t^i$ with $c_i \in K$, $\sup_i \{|c_i| \rho^i\} = 1$, and $|c_i| \rho^i \to 0$ as $i \to -\infty$. There must exist an index i with $|c_i| \rho^i = 1$; we can then choose j for which $|u_j \partial_j (c_i x)| = |c_i x|$. Then $|u_j \partial_j (z)| = |z|$, as desired. □

Lemma 5. *Under Hypothesis 1, view F_1 as a differential field equipped with $\partial_0 = \partial/\partial t$ together with $\partial_1, \dots, \partial_m$. Then for any finite irreducible differential module V over F_1 with intrinsic generic radius of convergence not equal to 1, there exists $j \in \{1, \dots, m\}$ such that ∂_j is dominant on V.*

Proof. By the proof of [**1**, Theorem 7.5.3], for some finite extension F'_1 of F_1, for each irreducible component W of $V \otimes_{F_1} F'_1$, there exists $r \in F'_1$ such that $E(r) \otimes W$ has intrinsic generic radius of convergence strictly greater than W. (Here $E(r)$ is the differential module with one generator $\boldsymbol{v}$ for which $\partial_j(\boldsymbol{v}) = \partial_j(r)\boldsymbol{v}$ for all j.) In particular, we must have $|r| > 1$. Choose some such W and r; by Lemma 3 and Lemma 4, there exists an index $j \in \{1, \dots, m\}$ such that $|u_j \partial_j (r)| = |r|$. For this j, ∂_j is dominant on W, and hence on V. □

We see now that Theorem 2.7.6 holds under Hypothesis 1, as follows. In the notation of that theorem, by Lemma 5, for each irreducible component of $M \otimes F_1$, there is an index $j \in \{1, \dots, m\}$ for which ∂_j is dominant on the component. In case we can make a uniform choice of j (e.g. if $m = 1$), the claim follows from Theorem 2.7.4.

Otherwise, choose $\rho \in (0, 1)$ with $-\log \rho < f_i(M, 0)$, and let K' be the completion of $K(w_1, \dots, w_m, z)$ for the $(1, \dots, 1, \rho)$-Gauss norm, carrying the extra derivation $\partial_{m+1} = \partial/\partial z$. Let F'_1 be the completion of $K'(t)$ for the 1-Gauss norm. Using Taylor series, we define a continuous embedding of K into K' taking u_j to $u_j(1 + w_j z)$ for $j = 1, \dots, m$; we similarly embed F_1 into F'_1. Using this embedding, form the base extension $M \otimes F'_1$, and write $\partial'_1, \dots, \partial'_m$ for the pullbacks of the actions of $\partial_1, \dots, \partial_m$ on $M \otimes F_1$. Then for $j = 1, \dots, m$, the action on ∂_j on $M \otimes F'_1$ is given by $(1 + w_j z)\partial'_j$, while the action of ∂_{m+1} on $M \otimes F'_1$ is given by $\sum_{j=1}^m w_j u_j \partial'_j$. We deduce that ∂_{m+1} is dominant on each component of $M \otimes F'_1$, so we may thus argue as in the previous paragraph.

We may also establish Theorems 2.7.12 and 2.7.13 under Hypothesis 1, by using the same construction as in the previous paragraph to reduce to Theorem 2.7.10.

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