10
Conclusions

This concludes our travel diary in the land of supersymmetric solitons in gauge theories. It is time to summarize the lessons.

Advances in supersymmetric solitons, especially in non-Abelian gauge theories, that have taken place since 1996, are impressive. In the bulk of this book we thoroughly discussed many aspects of the subject at a technical level. Important and relevant technical details presented above should not overshadow the big picture, which has been in the making since 1973. Sometimes people tend to forget about this big picture which is understandable: its development is painfully slow and notoriously difficult.

Let us ask ourselves: what is the most remarkable feature of quantum chromodynamics and QCD-like theories? The fact that at the Lagrangian level one deals with quarks and gluons while experimentalists detect pions, protons, glueballs and other color singlet states – never quarks and gluons – is the single most salient feature of non-Abelian gauge theories at strong coupling. Color confinement makes colored degrees of freedom inseparable. In a bid to understand this phenomenon Nambu, 't Hooft and Mandelstam suggested in the mid 1970s (independently but practically simultaneously) a "non-Abelian dual Meissner effect." At that time their suggestion was more of a dream than a physical scenario. According to their vision, "non-Abelian monopoles" condense in the vacuum resulting in formation of "non-Abelian chromoelectric flux tubes" between color charges, e.g. between a probe heavy quark and antiquark. Attempts to separate these probe quarks would lead to stretching of the flux tubes, so that the energy of the system grows linearly with separation. That's how linear confinement was visualized. However, at that time the notions of non-Abelian flux tubes and non-Abelian monopoles (let alone condensed monopoles in non-Abelian gauge theories) were nonexistent. Nambu, 't Hooft and Mandelstam operated with nonexistent objects.

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One may ask where these theorists got their inspiration from. There was one physical phenomenon known since long ago and well understood theoretically which yielded a rather analogous picture.

In 1933 Meissner discovered that magnetic fields could not penetrate inside superconducting media. The expulsion of the magnetic fields by superconductors goes under the name of the Meissner effect. Twenty years later Abrikosov posed the question: "What happens if one immerses a magnetic charge and an anticharge in type-II superconductors [which in fact he discovered]?" One can visualize a magnetic charge as an endpoint of a very long and very thin solenoid. Let us refer to the N endpoint of such a solenoid as a positive magnetic charge and the S endpoint as a negative magnetic charge.

In the empty space the magnetic field will spread in the bulk, while the energy of the magnetic charge-anticharge configuration will obey the Coulomb $1/r$ law. The force between them will die off as $1/r^2$.

What changes if the magnetic charges are placed inside a large type-II superconductor?

Inside the superconductor the Cooper pairs condense, all electric charges are screened, while the photon acquires a mass. According to modern terminology, the electromagnetic U(1) gauge symmetry is Higgsed. The magnetic field cannot be screened in this way; in fact, the magnetic flux is conserved. At the same time the superconducting medium does not tolerate the magnetic field.

The clash of contradictory requirements is solved through a compromise. A thin tube is formed between the magnetic charge and anticharge immersed in the superconducting medium. Inside this tube superconductivity is ruined – which allows the magnetic field to spread from the charge to the anticharge through this tube. The tube transverse size is proportional to the inverse photon mass while its tension is proportional to the Cooper pair condensate. These tubes go under the name of the Abrikosov vortices. In fact, for arbitrary magnetic fields he predicted lattices of such flux tubes. A dramatic (and, sometimes, tragic) history of this discovery is nicely described in Abrikosov's Nobel Lecture.

Returning to the magnetic charges immersed in the type-II superconductor under consideration, one can see that increasing the distance between these charges (as long as they are inside the superconductor) does not lead to their decoupling – the magnetic flux tubes become longer, leading to a linear growth of the energy of the system.

The Abrikosov vortex lattices were experimentally observed in the 1960s. This physical phenomenon inspired Nambu, 't Hooft and Mandelstam's ideas on non-Abelian confinement. Many people tried to quantify these ideas. The first breakthrough, instrumental in all current developments, came 20 years later, in the form of the Seiberg–Witten solution of $\mathcal{N} = 2$ super-Yang–Mills. This theory has eight supercharges which makes dynamics quite "rigid" and helps one to find the full analytic solution at low energies. The theory bears a resemblance to quantum chromodynamics, sharing common "family traits." By and large, one can characterize it as QCD's "second cousin."

An important feature which distinguishes it from QCD is the adjoint scalar field whose vacuum expectation value triggers the spontaneous breaking of the gauge symmetry $SU(2) \to U(1)$. The 't Hooft–Polyakov monopoles ensue. They are readily seen in the quasiclassical domain. Extended supersymmetry and holomorphy in certain parameters which is associated with it allows one to analytically continue in the domain where the monopoles become light – eventually massless – and then condense after a certain small deformation breaking $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ is introduced. After that, at a much lower scale the (dual) U(1) gauge symmetry breaks, so that the theory is fully Higgsed. Electric flux tubes are formed.

This was the first ever demonstration of the dual Meissner effect in non-Abelian theory, a celebrated analytic proof of linear confinement, which caused much excitement and euphoria in the community.

It took people three years to realize that the flux tubes in the Seiberg–Witten solution are not those we would like to have in QCD.[1] Hanany, Strassler and Zaffaroni, who analyzed the chromoelectric flux tubes in the Seiberg–Witten solution in 1997, showed that these flux tubes are essentially Abelian (of the Abrikosov–Nielsen–Olesen type) so that the hadrons they would create would not have much in common with those in QCD. The hadronic spectrum would be significantly richer. And, say, in the SU(3) case, three flux tubes in the Seiberg–Witten solution would not annihilate into nothing, as they should in QCD ...

Ever since, searches for non-Abelian flux tubes and non-Abelian monopoles continued, with a decisive breakthrough in 2003. By that time the program of finding field-theory analogs of all basic constructions of string/D-brane theory was in full swing. BPS domain walls, analogs of D branes, had been identified in supersymmetric Yang–Mills theory. It had been demonstrated that such walls support gauge fields localized on them. BPS-saturated string-wall junctions had been constructed. And yet, non-Abelian flux tubes, the basic element of the non-Abelian Meissner effect, remained elusive.

They were first found in U(2) super-Yang–Mills theories with extended supersymmetry, $\mathcal{N} = 2$, and two matter hypermultiplets. If one introduces a non-vanishing Fayet–Iliopoulos parameter ξ the theory develops isolated quark vacua, in which the gauge symmetry is fully Higgsed, and all elementary excitations are

[1] The Seiberg–Witten strings hopefully belong to the same universality class as the QCD strings, but this is impossible to prove with existing knowledge.

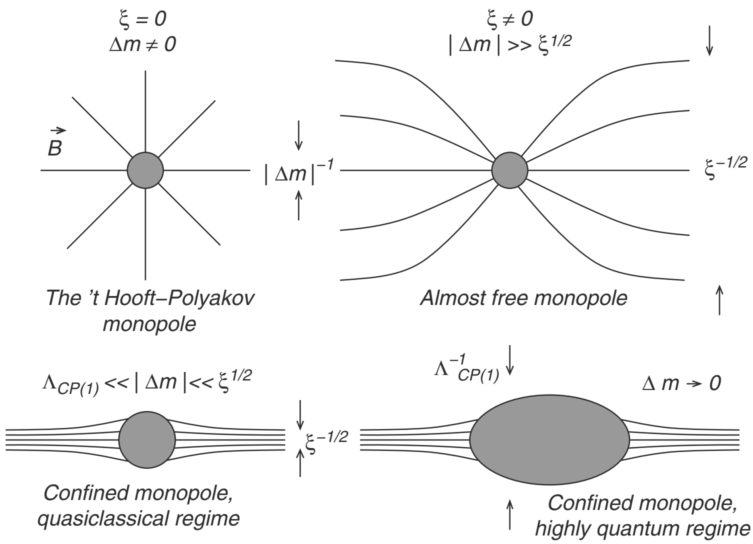

Figure 10.1. Various regimes for monopoles and strings in the simplest case of two flavors.

massive. In the general case, two matter mass terms allowed by $\mathcal{N} = 2$ are unequal, $m_1 \neq m_2$. There are free parameters whose interplay determines dynamics of the theory: the Fayet–Iliopoulos parameter ξ, the mass difference Δm and a dynamical scale parameter Λ, an analog of the QCD scale $\Lambda_{\rm QCD}$. Extended supersymmetry guarantees that some crucial dependences are holomorphic, and there is no phase transition.

As various parameters vary, this theory evolves in a very graphic way, see Fig. 10.1 which is almost the same as Fig. 4.3 (the first stage of unconfined 't Hooft–Polyakov monopole is added in the left upper corner). At $\xi = 0$ but $\Delta m \neq 0$ (and $\Delta m \gg \Lambda$) it presents a very clear-cut example of a model with the standard 't Hooft–Polyakov monopole. The monopole is free to fly – the flux tubes are not yet formed.

Switching on $\xi \neq 0$ traps the magnetic fields inside the flux tubes, which are weak as long as $\xi \ll \Delta m$. The flux tubes change the shape of the monopole far away from its core, leaving the core essentially intact. Orientation of the chromomagnetic field inside the flux tube is essentially fixed. The flux tubes are Abelian.

With $|\Delta m|$ decreasing, fluctuations in the orientation of the chromomagnetic field inside the flux tubes grow. Simultaneously, the monopole seen as the string junction, loses resemblance with the 't Hooft–Polyakov monopole. It acquires a life of its own.

Finally, in the limit $\Delta m \to 0$ the transformation is complete. A global SU(2) symmetry restores in the bulk. Orientational moduli develop on the string world

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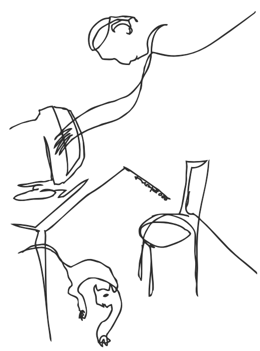
sheet making it non-Abelian. The junctions of degenerate strings present what remains of the monopoles in this highly quantum regime. It is remarkable that, despite the fact we are deep inside the highly quantum regime, holomorphy allows one to exactly calculate the mass of these monopoles.

What remains to be done? The most recent investigations zero in on $\mathcal{N} = 1$ theories, which are much closer relatives of QCD than $\mathcal{N} = 2$.

And then, $\mathcal{N} = 0$ theories – sister theories of QCD – loom large ...



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