

TOPOLOGY OF INTERCONNECTION NETWORKS WITH GIVEN DEGREE AND DIAMETER

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This thesis deals with the design of optimal interconnection networks. Optimality is interpreted as the largest possible number of nodes in the network, under given constraints on the number of connections attached to a node (*degree of the node*), and on the length of shortest paths between any two nodes (*diameter of the network*). Any interconnection network with bidirectional channels can be modelled by an undirected graph referred to as the ‘topology’ of the network. In graph theory this interpretation of optimality is known as the degree/diameter problem [8]. Under these constraints, there is an upper bound $M_{\Delta, D}$ on the maximum number of nodes that a network of maximum degree Δ and diameter D can have, which is known as the Moore bound [2, 8]. For bipartite network topology, the bipartite Moore bound $M_{\Delta, D}^b$ is defined in a similar way to the general Moore bound [2, 8].

I address the degree/diameter problem for both general graphs and bipartite graphs. Interconnection networks modeled by (bipartite) Moore graphs—namely, (bipartite) graphs attaining the (bipartite) Moore bound—are optimal.

Studies on Moore graphs and bipartite Moore graphs proved their rarity [1, 6, 12]. Consequently, research efforts to search for such optimal graphs fall into two main streams.

- (i) Lowering the upper bound on the number of vertices of a graph of maximum degree Δ and diameter D by proving the nonexistence or otherwise of graphs whose number of vertices is close to the corresponding Moore bounds.
- (ii) Increasing the lower bound on the number of vertices of a graph of maximum degree Δ and diameter D by constructing largest known graphs of maximum degree Δ and diameter D .

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(Bipartite) graphs of given maximum degree Δ , diameter D and order $(M_{\Delta,D}^b - \epsilon)$ $M_{\Delta,D} - \epsilon$ are called (*bipartite*) $(\Delta, D, -\epsilon)$ -graphs.

I also investigate regular topologies with the smallest possible number of vertices given degree and *girth* (the length of a shortest cycle in the graph): the degree/girth problem [5]. I consider only the case of even girth, in which case the bipartite Moore bound is the best known lower bound on the minimum number of vertices of such topologies. Regular graphs of degree d , girth g and order $M_{d,g/2}^b + \epsilon$ are called $(d, g, +\epsilon)$ -graphs.

The following results are the cornerstones of this thesis.

Large graphs of diameter 6. I produce a family of large compound graphs of diameter 6 (see [10]). Several members of this family are currently the largest known graphs for their respective maximum degree.

Complete catalogue of $(3, D, -4)$ -graphs. In the context of the degree/diameter problem for general graphs, I complete the characterization of all $(3, D, -\epsilon)$ -graphs with $D \geq 2$ and $0 \leq \epsilon \leq 4$, the main result being a nonexistence proof of $(3, D, -4)$ -graphs for $D \geq 5$ (see [7, 11]).

Bipartiteness of $(3, g, +4)$ -graphs for even g . I study the degree/girth problem for even girth, obtaining as the main result that if $(3, g, +4)$ -graphs with $g \geq 12$ exist then they must be bipartite. Furthermore, I conjecture that such graphs do not exist.

Bipartite $(\Delta, 3, -2)$ -graphs. I study the degree/diameter problem for bipartite graphs. More precisely, I derive several necessary conditions for the existence of bipartite $(\Delta, 3, -2)$ -graphs, and prove the uniqueness of the two known such graphs (see [4]).

Bipartite $(\Delta, D, -2)$ -graphs. This research follows up previous studies concerning the bipartite version of the degree/diameter problem. Here I prove that bipartite $(\Delta, D, -2)$ -graphs with $\Delta \geq 3$ and $D \geq 4$ do not exist (see [3, 9]).

Finally, I present a number of open problems and conjectures, providing scope for future research in the area of network design.

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