

EVOLUTION OF ORBITS AND INTERSECTION CONDITIONS WITH THE EARTH OF THE GEMINID AND QUADRANTID METEOR STREAMS

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The Geminids and Quadrantids belong to the most active annual meteor showers. The distinct difference between the orbit sizes of large and small meteoroids in these streams has been shown. From this, the estimated age of the streams greatly exceeds the time since their discovery in the XIX century (Geminids in 1862, Quadrantids in 1830) (Lovell 1954, Hindley 1972).

This paper presents the results of a study of the orbital evolution of the Geminids and Quadrantids and the intersection conditions of the streams with the Earth taking into account nongravitational effects, as well as the perturbations from six planets.

The evolution of the orbit as influenced by Poynting-Robertson effect and its corpuscular analogue taking into account the changes of meteoroid mass due to evaporation and sputtering can be represented by the system of differential equations (Whipple 1955, Dobrovolsky et al. 1973):

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{2} \sqrt[3]{\frac{9\pi}{16\delta^2 m}} \frac{W(2+3e^2)}{a(1-e^2)^{1.5}}, \\ \frac{de}{dt} &= -\frac{5}{4} \sqrt[3]{\frac{9\pi}{16\delta^2 m}} \frac{We}{a^2(1-e^2)^{0.5}}, \\ \frac{dm}{dt} &= -\sqrt{\frac{9\pi m^2}{16\delta^2}} \frac{1}{\pi a^2(1-e^2)^{0.5}} \left(\pi B + \frac{4M}{N_0} \int_0^\pi r^2 z d\nu \right); \end{aligned} \tag{1}$$

where a , e are the orbital semimajor axis and eccentricity of the meteoroid, having mass m , density δ , molecular weight M and latent heat of vaporization L , $W = 8.44 \cdot 10^8$ kg/s is the mass change rate of the Sun due to electromagnetic and corpuscular radiation, B is the rate of sputtering at the distance of 1 AU, $Z = Z(r, M; L)$ is the rate of evaporation according to Huebner (1970), N_0 is the Avogadro constant, r the radius-vector and ν the true anomaly of the meteoroid.

From photographic and radar observations it has been shown that the difference of the orbital semimajor axis of large and small Geminid meteoroids is equal to 0.1 AU (Jones 1978) and that for Quadrantid meteoroids is 0.2 AU (Hindley 1972). Considering these differences as a

result of nongravitational effects it is possible to calculate the time required for generating them, i.e. the age of the streams.

Taking the Geminid orbital elements according to Cook (1973) and solving the equations (1) by a numerical method we conclude that a stone particle with $\delta = 1-3.5 \text{ Mg/m}^3$, $M = 24$ and $L = 7 \text{ MJ/kg}$ is evaporated and sputtered so intensively that its mass decreases from 10^{-3} kg to 10^{-16} kg during 15-50 years (Figure 1). During this time the orbital semi-major axis changes only 10^{-4} AU . In the case of an iron particle ($\delta = 7.6 \text{ Mg/m}^3$, $M = 56$, $L = 6.7 \text{ MJ/kg}$) the evaporation and sputtering can be neglected because the particle mass changes only 1% during 4000 years. According to equations (1) it requires 15000 years to change the orbital semimajor axis of a Geminid iron meteoroid of mass 10^{-6} kg by 0.1 AU.

On the basis of the above one can say that the lifetime of stone particles of mass 10^{-3} kg on the Geminid orbit (also the Quadrantid orbit if we take into account secular perturbations, Figure 1) would not exceed 50 years. Therefore we can assume that Geminids and Quadrantids are streams of particles which have great values of δ , M and L (iron, nickel, manganese, silicon and etc.).

Since according to equations (1) for meteor stream ages we get rather large values (15000 years for Geminids and 60000 years for Quadrantids) it is necessary to examine the evolution of stream orbits for a long time under the perturbing action of the planets.

The differential equations for determining secular perturbations of orbital elements may be written in the form (Gorjachev 1937):

$$\begin{aligned} \frac{de}{dt} &= \frac{m'n a \sqrt{1-e^2}}{2\pi} \int_0^{2\pi} [Sr \sin v + Tr(\cos E + \cos v)] dE, \\ \frac{di}{dt} &= \frac{m'n}{2\pi \sqrt{1-e^2}} \int_0^{2\pi} W r^2 \cos(\omega + v) dE, \\ \frac{d\Omega}{dt} &= \frac{m'n}{2\pi \sqrt{1-e^2} \sin i} \int_0^{2\pi} W r^2 \sin(\omega + v) dE, \\ \frac{d\omega}{dt} &= \frac{m'n a \sqrt{1-e^2}}{2\pi e} \int_0^{2\pi} [-Sr \cos v + T(1 + \frac{r}{p}) \sin v] de - \cos i \frac{d\Omega}{dt}; \end{aligned} \tag{2}$$

where a , e , i , Ω , ω are elements and p is a parameter of the orbit of the disturbing body, n , r , v and E are its average motion, radius-vector, true and eccentric anomaly; S , T , W the components of disturbing accelerations, m' the mass of the perturbing planet.

Calculations of secular perturbations from six planets (Mercury-Saturn) were carried out according to the Halphen-Gorjachev method (Gorjachev 1937). The results are presented in tables 1 and 2, where the orbital elements are given for corresponding times (1950 is taken as zero). The intersection of a stream with the Earth is possible when the radius-vector of the ascending r_{as} or descending r_{des} node is equal to 1 AU. From tables 1 and 2 it follows that the Geminid stream meets the requirements of $r \approx 1 \text{ AU}$ in -19500, 0 and 12000 years in descending

Table 1. The evolution of the Geminid meteor stream orbit.

$t \cdot 10^{-3}$ yrs	e	q	i	Ω	ω
-20	0.88	0.16	29	185	28
-19	0.90	0.14	23	172	41
-18	0.90	0.13	17	145	67
-17	0.90	0.13	16	104	108
-16	0.90	0.14	22	75	136
-15	0.89	0.16	29	61	149
-14	0.87	0.18	35	53	157
-13	0.85	0.20	40	49	163
-12	0.84	0.22	43	46	168
-11	0.82	0.24	45	43	172
-10	0.82	0.25	46	41	176
-9	0.82	0.25	46	40	181
-8	0.82	0.24	45	38	185
-7	0.84	0.22	43	36	189
-6	0.86	0.20	39	33	194
-5	0.87	0.17	34	28	199
-4	0.89	0.15	28	21	207
-3	0.90	0.13	20	6	222
-2	0.91	0.13	14	331	256
-1	0.90	0.13	16	285	301
0	0.90	0.14	24	261	324
1	0.88	0.16	31	250	336
2	0.86	0.19	37	244	342
3	0.84	0.21	41	240	348
4	0.83	0.24	44	237	352
5	0.82	0.25	46	235	356
6	0.81	0.26	46	233	1
7	0.82	0.25	46	232	5
8	0.83	0.24	45	230	10
9	0.84	0.22	43	227	14
10	0.86	0.19	39	224	19
11	0.88	0.16	34	218	25
12	0.90	0.14	27	209	34
13	0.91	0.12	20	191	52
14	0.91	0.12	16	153	89
15	0.91	0.12	19	113	127
16	0.90	0.13	27	93	146
17	0.89	0.15	34	84	156
18	0.87	0.18	40	78	162
19	0.85	0.20	44	75	167
20	0.84	0.22	46	72	171

Table 2. The evolution of the Quadrantid meteor stream orbit.

$t \cdot 10^{-2}$ yrs	e	q	i	Ω	ω
-20	0.94	0.20	54	94	13
-19	0.95	0.16	48	92	14
-18	0.96	0.12	39	89	16
-17	0.97	0.09	28	82	22
-16	0.98	0.08	16	61	41
-15	0.98	0.07	12	355	105
-14	0.97	0.08	21	315	144
-13	0.97	0.10	31	302	156
-12	0.96	0.12	40	296	160
-11	0.95	0.16	48	292	162
-10	0.93	0.20	53	290	163
-9	0.92	0.26	58	288	164
-8	0.89	0.32	61	287	164
-7	0.87	0.40	64	286	164
-6	0.84	0.48	66	286	164
-5	0.82	0.57	68	285	165
-4	0.79	0.65	69	284	165
-3	0.76	0.74	70	284	166
-2	0.72	0.85	72	284	166
-1	0.70	0.92	72	283	168
0	0.68	0.98	72	283	170
1	0.66	1.06	73	282	172
2	0.64	1.10	73	282	174
3	0.64	1.12	73	282	177
4	0.62	1.16	74	281	180
5	0.63	1.13	73	281	183
6	0.64	1.10	73	281	185
7	0.65	1.08	73	280	188
8	0.68	1.00	72	280	190
9	0.70	0.93	72	280	191
10	0.72	0.87	72	280	193
11	0.75	0.77	70	279	194
12	0.78	0.67	69	279	195
13	0.81	0.59	68	278	195
14	0.84	0.51	67	278	196
15	0.86	0.42	65	277	196
16	0.89	0.34	63	276	196
17	0.91	0.27	60	276	196
18	0.92	0.22	56	274	196
19	0.95	0.16	51	273	197
20	0.96	0.12	45	270	198

$t = 0$ corresponds to 1950.0

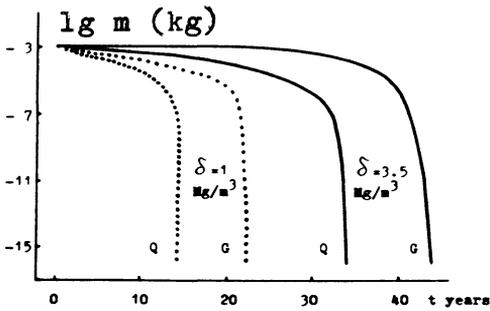


Fig. 1. Decrease of meteoroid mass due to vaporization and pulverization. G-Geminids, Q-Quadrantids 1500 years ago.

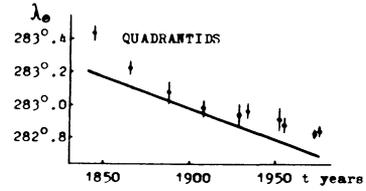


Fig. 4. Longitude of the Sun (λ_{\odot}) at the Quadrantid maximum: dots, according to observation (Hindley 1972) and, solid line, calculations of secular perturbations.

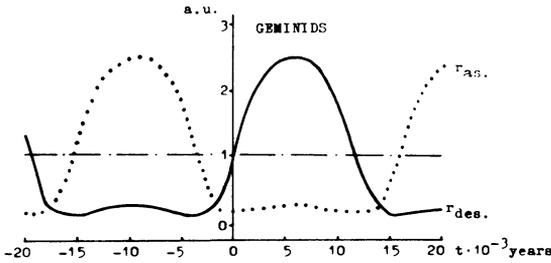


Fig. 2. Radius-vector of ascending (r_{as}) and descending (r_{des}) nodes of Geminids and Quadrantids versus time.

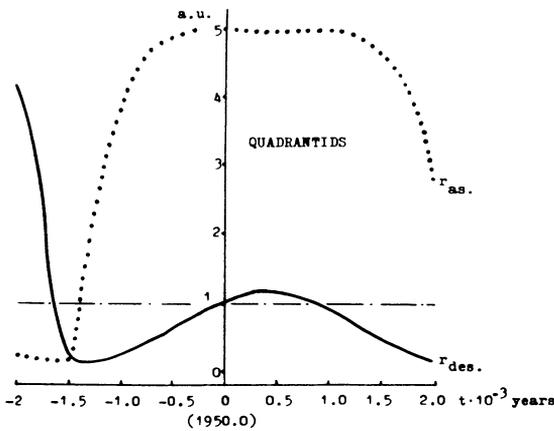
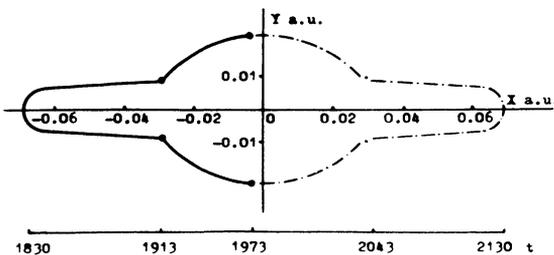


Fig. 3. Assumed section of Quadrantid stream with ecliptic plane. X-axis is directed towards the Sun, Y-axis is directed along the Earth's motion.



node and in -15500, -3500 and 16000 years in ascending node. For Quadrantids the condition $r \approx 1$ AU holds at -1350 and 2200 years in ascending node and in -1650, 0 and 850 years in descending node (Figure 2). The rate of change of r_{des} and r_{as} at these moments for both streams is very great, $\sim 5 \cdot 10^{-4}$ AU/year. According to Whipple and Wright (1954) the width of the Geminid stream is equal to 0.1 AU. If the stream width along the radius-vector of the descending node is the same order of magnitude, then the period of the Geminid shower activity is ~ 200 years and thus the appearance of the Geminid shower in 1862 can be explained by the disturbing action of planets.

We can make another independent estimate of the visibility period of the Geminids at present. If we take 1958 as the moment of Earth passage through the central part of the stream, because the Geminid meteor hourly rate was anomalously high in 1958 (Hajduk et al. 1974) and if the stream is considered symmetrical along r_{des} , then for the period of shower visibility we shall take $2 \cdot (1958 - 1862) = 192$ years, which agrees with the above mentioned estimate.

As to Quadrantids, it was found that in 1913 the stream width was 0.016 AU (Lovell 1954). At present Quadrantid activity lasts about 2.6 days (Cook 1973) which corresponds to a stream width of 0.045 AU along the Earth's orbit. According to the rate of change of r_{des} we may conclude that during the interval of 1913-1973 the stream displaced by 0.03 AU from the Sun with respect to the Earth's orbit at the ecliptic plane and the duration of shower activity has increased. From these data and taking into account the discovery of Quadrantids in 1830 we are able to construct a possible cross-section of the stream with the ecliptic plane (Figure 3).

Figure 4 shows the longitude of the Sun at the observed time of Quadrantid maximum activity in comparison with the results of calculated variations of the orbital longitude of the ascending node (table 2). The satisfactory accordance of calculations with observations allows us to confirm that the Earth crosses the stream approximately normally to the major axis of the stream cross-section with the ecliptic plane (Figure 3), i.e. the displacement of shower maximum activity is exactly due to the secular perturbations.

On account of the annual increase of r_{des} the Earth meets the meteoroids which are in more inner regions of the stream. If the displacement of meteor particles into the stream is a result of Poynting-Robertson effect and its corpuscular analogue then the increase of meteor hourly rates during recent years must be due mainly to more smaller particles, which has been confirmed by visual observations in Dushanbe. Moreover if the stream cross-section along r_{des} is 0.15 AU then the period of shower visibility is nearly equal to 300 years.

The analogous calculation of the period of Perseid visibility gives 20000 years.

According to table 2 we may assume that the Quadrantids could be observed at their ascending node about 1300 years ago and it is possible that the observations of shower No 30 at $\lambda_{\odot} = 127^{\circ}$ in 784 A.D. (Imoto and Hasegawa 1958) concerned them.

It should be noted that if secular perturbations of perihelion distance are considerable (as for Quadrantids) then the decrease rate of semimajor axis influenced by the Poynting-Robertson effect and its

corpuscular analogue are not constant (Figure 5). Therefore, in order to determine the stream age it is necessary to take into account the secular perturbations and nongravitational effects simultaneously, i.e. we must solve equations (1) and (2) jointly. Thus, taking into account that the smallest particles of the Geminid stream have a mass of 10^{-6} kg (Jones 1978), when $\delta = 0.3, 1$ and 7.6 Mg/m^3 we obtain 1600, 3500 and 19000 years respectively for the stream's age (assuming $M = 56$ and $L = 6.73 \text{ MJ/kg}$). In the Quadrantid stream the smallest particles have a mass equal to $2.4 \cdot 10^{-7}$ kg which corresponds to a meteor of $6^{\text{m.5}}$ (Hindley 1972) and for the stream ages for the same meteoroid densities we obtain 3000, 10000 and 40000 years.

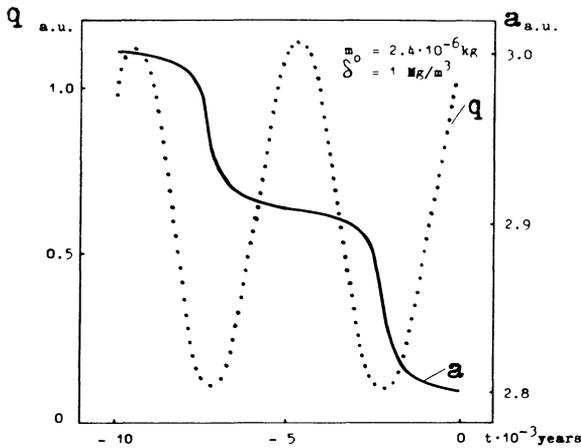


Fig. 5. Variations of semimajor axis a and perihelion distance q under the simultaneous influence of Poynting-Robertson effect and secular perturbations versus time.

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