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BOOK REVIEWS

ARNOLD, V. I., *Catastrophe theory* (translated by R. K. Thomas, Springer-Verlag, Berlin-Heidelberg-New York 1984), ix + 79 pp, DM 19.

This is a translation of the Russian original (1981). The preface claims that it is written for "readers having minimal mathematical background but the reader is assumed to have an inquiring mind". It seems clear to me that every mathematician would find something interesting and useful in this book and would gain an excellent overview of a variety of modern geometric methods by reading it. I would find it interesting to discover what a reader with a "minimal mathematical background" would get out of it—surely something quite different from that gained by a mathematician.

The publicity generated by catastrophe theory has done a great deal to bring mathematical methods to the attention of a wide audience. Arnold's booklet could help to deepen and broaden this interest as little "mathematical technology" is needed to read most of it and one can see how new ideas can be applied to enable one to get a better understanding of problems that are both interesting and difficult. Mathematicians critical of catastrophe theory should find some reassurance in these pages—Arnold manages to take a moderate stance about the controversies but without loss of enthusiasm. Too often the critics have been rather narrow in stressing the lack of precision in the applications of catastrophe theory and have forgotten that a mathematical model can have other virtues (an excellent example is given by M. W. Hirsch in Sections 4 and 5 of his article in Volume 11 (1984) of the *Bulletin of the American Mathematical Society*).

Arnold's book consists of fifteen short chapters. The first two are in sharp contrast to each other. The first explains the background and mentions the controversies. The second is an account of H. Whitney's 1955 paper on mappings from $\mathbb{R}^2$ to $\mathbb{R}^2$, whose results can be regarded as one of the starting points of the mathematical apparatus behind catastrophe theory. The next chapter contains a witty "application" to the work of a scientist—a catastrophe can result if enthusiasm grows faster than technique! Next, there is a heuristic account of Zeeman's catastrophe machine and this leads naturally to the next two chapters about the bifurcations and loss of stability of an equilibrium. The pace is quite fast by now and much of the material involves topics in the geometric theory of ordinary differential equations with emphasis laid on the work of the Russian school that successfully developed many of Poincaré's ideas (Andronov *et al.* on oscillation theory, Anosov and Sinai on structural stability, with applications by Arnold himself to hydrodynamic stability; there is also a brief account of very recent Russian work on these topics). The Lorentz equations and the work of Smale are also mentioned.

The seventh chapter discusses stability boundaries and brings in Arnold's "Principle of the Fragility of Good Things"—a seemingly naive but exciting concept that is bound to cast new light on a number of interesting phenomena. There follow good expositions (with clear diagrams) of several more conventional topics in singularity theory such as caustics and wave fronts which are of special interest because of their ubiquity in real life. There are also interesting historical and philosophical comments such as the remark that the reason for the previous lack of a satisfactory theory for some of these phenomena, which have been well-known for centuries, is that modern mathematical tools are essential to their understanding. Arnold also mentions the amusing theory proposed by V. M. Zakalyukin that flying saucer "sightings" may really be due to the appearance of a caustic when light passes through dust or fog.

Chapter Nine gives a description (due to Y. B. Zel'dovich) of how a uniform distribution of matter in space can cluster; this might explain clustering into galaxies, and to some extent his theory agrees with astronomical observations. In contrast, the next topic is perhaps the most familiar of all aspects of singularities—the theory of maxima and minima, but again Arnold has something new to say. This leads on to control theory and the study of the singularities of accessibility boundaries. An example given of control theory is tacking in sailing; as elsewhere, Arnold manages to find very simple but illuminating applications for the mathematics that he is discussing. Chapters on the projections of smooth surfaces onto planes and on the study of shortest paths that bypass obstacles illustrate other uses of singularity theory.

The penultimate chapter is on symplectic and contact geometries; these are topics that Arnold himself has done so much to bring to the forefront of modern geometry and its applications. These closely related topics have their roots firmly based in nineteenth-century work on analytical dynamics, a development culminating in E. T. Whittaker's influential treatise. In recent years the subject has been clarified enormously by the use of the axiomatic method (Arnold mentions Bertrand Russell's remark that this method has advantages similar to those that stealing has over honest work). Not only has Arnold given a masterly account of classical mechanics (in his book *Mathematical methods of classical mechanics*) from this viewpoint, but he has also developed variants of singularity theory that exploit the extra structure given by symplectic and contact geometries and has shown their relevance to the understanding of several interesting phenomena. These general ideas may even have more lasting significance than singularity theory itself; Arnold certainly believes in their importance as the final paragraph of this fourteenth chapter shows clearly.

The final chapter entitled "The mystics of catastrophe theory" begins with a slightly critical account of Thom's philosophy but ends with a more common brand of mathematical mysticism—why do the Dynkin diagrams appear in so many diverse branches of mathematics? The excuse for bringing this question in here is that Dynkin diagrams do appear in singularity theory.

The book is one of a rare kind amongst mathematics books: it is written for a general audience but does not talk down to the reader; additionally, the professional mathematician can return to it time and time again both for inspiration and for information about current mathematics. Arnold enjoys both doing mathematics and writing about it in all its forms and is fascinated by its internal mysteries and its applications; this enthusiasm is obvious on every page. It is something of a pity that the publishers forgot to proof-read it before printing; however, there is a loose errata sheet with further references. In many places the book can whet the appetite for more but tantalisingly often there are no adequate references. Fortunately the book *Singularities of differentiable maps,* Volume 1, by V. I. Arnold, S. M. Gusein-Zade and A. N. Varchenko (Birkhäuser 1985) has appeared in the meantime and contains mathematical details on most of the topics discussed in the present book as well as many references to the work of Arnold's students and collaborators.

Finally, I would like to say that I recommend that everyone should spend at least an evening enjoying this book.

E. G. REES

Dubrovin, B. A., Fomenko, A. T. and Novikov, S. P., *Modern geometry—methods and applications. Part 2: The geometry and topology of manifolds* (translated by R. G. Burns, Graduate Texts in Mathematics 104, Springer-Verlag, Berlin-Heidelberg-New York 1985), xv+430 pp., DM 158.

This book is Part Two of a trilogy whose aim is no less than an exposition of modern geometry. The subject matter of this volume is the geometry and topology of manifolds, a quite enormous area, and the authors make no pretence at an encyclopedic treatment. Indeed, their philosophy is quite neatly encapsulated in the following quotation from their introduction. They claim to have striven to

> "minimise the degree of abstraction of the exposition and terminology, often sacrificing thereby some of the so-called 'generality' of statements and proofs: frequently an important result may be obtained in the context of crucial examples containing the whole essence of the