

# Notation Index: In Order of Appearance

- $(\mathcal{M}, \langle \cdot, \cdot \rangle)$  – Minkowski space, 5
- $\langle \cdot, \cdot \rangle$  – Minkowski metric, 5
- $g_{jk}$  – Minkowski metric in components, 5
- $L$  – light cone, 6
- $I$  – interior light cone, 6
- $J$  – closed light cone, 6
- $J^\vee, J^\wedge$  – closed future and past light cone, 6
- $I^\vee, I^\wedge$  – open future and past light cone, 6
- $\square$  – scalar wave operator, 7
- $\gamma^j$  – Dirac matrix, 8
- $\not{A}, \not{B}$  – Feynman dagger, 8
- $\langle \cdot | \cdot \rangle, \langle \cdot | \cdot \rangle_x$  – spin inner product, 9, 70, 111
- $(\cdot | \cdot), (\cdot | \cdot)_m$  – scalar product on Dirac solutions in Minkowski space, 10, 280
- $B$  – external potential, 10
- $H$  – Dirac Hamiltonian, 11, 87
- $\langle \cdot | \cdot \rangle$  – spacetime inner product, 11, 78, 259
- $\Gamma$  – pseudo-scalar matrix, 13, 72
- $\epsilon_{jklm}$  – totally anti-symmetric symbol, 13, 178
- $\chi_L, \chi_R$  – chiral projectors, 13
- $S\mathcal{M}$  – spinor bundle in Minkowski space, 14
- $C_{sc}^\infty(\mathcal{M}, S\mathcal{M})$  – spatially compact spinorial wave functions, 14, 87, 297
- $C_0^\infty(\mathcal{M}, S\mathcal{M})$  – spinorial wave functions of compact support, 15
- $d(x, y)$  – metric, 21
- $\mathcal{O}$  – topology, 22, 23
- $\bar{A}$  – closure of  $A$ , 23
- $\overset{\circ}{A}$  – interior of  $A$ , 23
- $\|\cdot\|$  – norm, 24
- $\langle \cdot | \cdot \rangle$  – complex scalar product, 25
- $\delta_{ij}$  – Kronecker delta, 26
- $(L(V, W), \|\cdot\|)$  – Banach space of linear bounded operators, 27
- $\|\cdot\|, \|\cdot\|_{\mathcal{H}}$  – sup-norm, operator norm, 27, 107
- $A^*$  – adjoint of operator, 28
- $I^\perp$  – orthogonal complement of  $I \subset \mathcal{H}$ , 29
- $\mathfrak{M}$  –  $\sigma$ -algebra of measurable sets, 30
- $L^p(\mathcal{F}, d\rho)$  –  $L^p$ -spaces with  $1 \leq p \leq \infty$ , 32
- $\langle \cdot | \cdot \rangle_{L^2(\mathcal{F}, d\rho)}$  –  $L^2$ -scalar product, 32
- $\chi_A$  – characteristic function of  $A$ , 32
- $\|f\|_{p,q}$  – Schwartz norm of  $f$ , 36
- $\mathcal{S}(\mathbb{R}^n)$  – Schwartz space, 37
- $\mathcal{S}'(\mathbb{R}^n)$  – space of tempered distributions, 37
- $\delta$  – (Dirac's)  $\delta$  distribution, 38
- $\mathcal{F}, \mathcal{F}^*$  – Fourier transform, 39
- $f * g$  – convolution of Schwartz functions, 41
- $f * T$  – convolution of Schwartz function with tempered distribution, 42
- $\mathcal{D}(\mathbb{R}^n)$  – space of test functions, 42
- $\mathcal{D}'(\mathbb{R}^n)$  – space of distributions, 42
- $T_p\mathcal{M}$  – tangent space of  $\mathcal{M}$  at  $p$ , 45
- $T\mathcal{M}$  – tangent bundle of  $\mathcal{M}$ , 47
- $\Theta$  – Heaviside function, 54
- $(\mathcal{M}, g)$  – Lorentzian manifold, 68
- $T_p\mathcal{M}$  – tangent space at  $p$ , 68
- $U(2, 2)$  – group of isometries of the spin inner product, 70
- $\nabla$  – Levi-Civita connection, 73
- $D$  – spin derivative, 75
- $(\cdot | \cdot)_{\mathcal{N}}$  – scalar product on Dirac solutions in curved spacetime, 78
- $S\mathcal{M}$  – spinor bundle, 84
- $(\mathcal{H}, \langle \cdot | \cdot \rangle_{\mathcal{H}})$  – Hilbert space of causal fermion system, 102
- $n$  – spin dimension, 102
- $\mathcal{F}$  – set of operators of causal fermion system, 102
- $\rho$  – measure on  $\mathcal{F}$  of causal fermion system, 102
- $(\mathcal{H}, \mathcal{F}, \rho)$  – causal fermion system, 102
- $M$  – spacetime of causal fermion system, 102
- $\mathfrak{R}_\varepsilon$  – regularization operator on scale  $\varepsilon$ , 104, 367
- $F^\varepsilon(x)$  – local correlation operator at  $x \in \mathcal{M}$  with regularization on scale  $\varepsilon$ , 105
- $F_*\mu$  – push-forward measure, 105
- $\mathcal{L}(x, y)$  – causal Lagrangian, 106
- $S(\rho)$  – causal action, 106
- $\mathcal{T}(\rho)$  – functional in boundedness constraint, 106
- $\text{tr}$  – trace of linear operator, 106
- $\lambda_1^{xy}, \dots, \lambda_{2n}^{xy}$  – nontrivial eigenvalues of  $xy$ , 108
- $\mathcal{C}$  – time direction functional, 108

- $S_x := x(\mathcal{H})$  – spin space, 109
- $(S_x M, \prec, \cdot, \succ_x)$  – spin space, 109, 111
- $\pi_x$  – orthogonal projection on spin space, 109
- $P(x, y)$  – kernel of fermionic projector, 109
- $\text{Tr}_{S_x}$  – trace on spin space, 110
- $A_{xy}$  – closed chain, 110
- $\lambda_1^{xy}, \dots, \lambda_{2n}^{xy}$  – nontrivial eigenvalues of  $xy$ , 111
- $\psi^u$  – physical wave function of  $u \in \mathcal{H}$ , 111
- $\Psi(x)$  – wave evaluation operator at  $x \in M$ , 112
- $C^0(M, SM)$  – continuous wave functions in spacetime  $M$ , 131
- $\mathcal{L}_\kappa$  –  $\kappa$ -Lagrangian, 144
- $\mathcal{F}^{\text{reg}}$  – regular points of  $\mathcal{F}$ , 144
- $\ell$  – integrated Lagrangian, 148
- $s$  – Lagrange parameter in Euler–Lagrange equations, 148
- $\nabla_u$  – jet derivative in the direction of  $u$ , 151
- $\mathfrak{J}$  – space of smooth jets, 151
- $\mathfrak{M}^{\mathcal{G}}$  – equivariant measures under symmetry group  $\mathcal{G}$ , 152
- $\Delta$  – linearized field operator, 159
- $\text{div}$  – divergence in smooth spacetime, 163
- $\Gamma$  – space of smooth jets without scalar component, 166
- $\Phi_\tau$  – variation of spacetime, 173
- $\gamma_\rho^\Omega$  – conserved one-form, 176
- $I_{k+1}$  – general conserved surface layer integral, 177
- $\mu(\mathbf{v}, x)$  – boundary measure induced by inner solution  $\mathbf{v}$ , 178
- $\langle \cdot, \cdot \rangle_\rho^\Omega$  – commutator inner product, 180
- $Q^{\text{dyn}}(x, y)$  – kernel in the dynamical wave equation, 181
- $\sigma_\rho^\Omega(\cdot, \cdot)$  – symplectic form on linearized solutions, 182
- $(\cdot, \cdot)_\rho^\Omega$  – surface layer inner product, 183
- $\gamma^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho)$  – nonlinear surface layer integral, 184
- $\nu$  – correlation measure, 185
- $Z^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho)$  – partition function, 186, 386
- $Q(x, y)$  – kernel describing the first variation of causal Lagrangian, 188
- $L_\rho$  – positive operator arising from second scalar variations, 198
- $s_x$  – Euclidean sign operator, 202
- $T_x$  – tangent space of causal fermion system, 202
- $D_x$  – spin connection of causal fermion system, 205
- $\nabla_x$  – metric connection of causal fermion system, 205
- $\ll$  – absolute continuity of measures, 226
- $\mathfrak{m}^{(\ell)}$  –  $\ell$ th moment measure, 229
- $C_x$  – tangent cone at  $x \in M$ , 239
- $\mu_x$  – tangent cone measure at  $x \in M$ , 239
- $\tilde{k}_m$  – causal fundamental solution, 258
- $\tilde{s}_m^\vee, \tilde{s}_m^\wedge$  – causal Green’s operators, 258, 335
- $(\eta_t)_{t \in I}$  – local foliation inside  $U \subset M$ , 268
- $(\cdot, \cdot)^t$  – softened surface layer inner product, 270
- $\sigma^t(\cdot, \cdot)$  – softened symplectic form, 270
- $\mathfrak{J}^{\text{vary}}$  – space of jets used for varying the measure, 271, 353
- $\mathfrak{J}_x, \Gamma_x$  – jets at spacetime point  $x \in M$ , 272
- $\|\cdot\|^t$  – norm corresponding to softened surface layer inner product, 272
- $\langle \cdot, \cdot \rangle_{L^2(L)} - L^2$  – scalar product in lens-shaped region, 273
- $\bar{\mathfrak{J}}, \mathfrak{J}$  – jet spaces vanishing in future or past, 274
- $\mathcal{S}$  – fermionic signature operator, 281
- $C_{\text{sc}, 0}^\infty(\mathcal{M} \times I, S\mathcal{M})$  – space of families of spinorial wave functions, 283
- $(\cdot, \cdot)_I$  – scalar product on families of Dirac solutions, 283
- $\mathcal{H}^\infty$  – domain for mass oscillation property, 283
- $T$  – operator of multiplication by mass parameter, 283
- $\mathfrak{p}$  – operator of integration over mass parameter, 284
- $s_m(x, y)$  – Dirac Green’s operator in Minkowski space, 296
- $s_m^\vee, s_m^\wedge$  – causal Green’s operators in the vacuum, 297, 333, 335
- $k_m$  – causal fundamental solution, 297
- $U^{t', t}$  – time evolution operator, 298
- $\epsilon$  – sign function, 301
- $\Pi_\pm(\vec{k})$  – projections to Dirac solutions in momentum space, 301
- $\text{Pexp}$  – ordered exponential, 318
- $R_\lambda$  – resolvent of causal fundamental solution, 324
- $T_{m^2}(x, y)$  – Fourier transform of lower mass shell, 329
- $\mathcal{O}((y - x)^{2p})$  – order on the light cone, 332
- $T^{(n)}(x, y)$  – singular factors in light-cone expansion, 333
- $S_{m^2}^\vee, S_{m^2}^\wedge$  – causal Green’s operators of Klein–Gordon equation, 334
- $B = \mathcal{B} - m$  – external potential combined with the mass, 335
- $S^{(l)}$  – mass expansion of  $S_a$ , 335
- $\text{WF}(\phi)$  – wave front set of  $\phi$ , 339
- $\mathfrak{J}^{\text{diff}}, \Gamma^{\text{diff}}$  – Space of jets for which  $\ell$  is differentiable, 353
- $\mathfrak{J}^{\text{test}}$  – space of test jets, 353
- $\xi_{[p]}^{(n)}$  – ultraviolet regularized factor  $\xi$ , 368
- $P^\varepsilon(x, y)$  – regularized kernel of fermionic projector, 368
- $z_{[p]}^{(n)}$  – abbreviation for  $(\xi_{[p]}^{(n)})^2$ , 369
- $T_{[p]}^{(n)}$  – ultraviolet regularized  $T^{(n)}$ , 369

$T_o^{(n)}$  – stands for  $T_{\{p\}}^{(n)}$  or  $T_{[p]}^{(n)}$ , 369

deg – degree on light cone, 369

$L$  – degree of simple fraction, 370

$c_{\text{reg}}$  – regularization parameter,  
370

$\nabla$  – derivation on the light cone, 370

$\ell_{\text{macro}}$  – length scale of macroscopic physics,  
371

$\Psi(x)$  – wave evaluation operator at  $x \in M$ ,  
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