SCHOFIELD, A. H., *Representations of rings over skew fields* (London Mathematical Society Lecture Note Series 92, Cambridge University Press 1985), xii+223 pp., £12.95.

There is a large literature on the study of finite-dimensional representations of rings over commutative fields and the study of such representations is of crucial importance in many algebraic theories. In contrast, little is known about the possible finite-dimensional representations of rings over skew fields (i.e. fields where multiplication is not required to be commutative). The present author addresses this problem boldly by giving a classification of all possible homomorphisms from an arbitrary ring to matrix rings over skew fields. Important earlier work that the author builds on includes P. M. Cohn's work on universal localization and the coproduct theorems of G. M. Bergman.

This book is divided into two parts. In the first part (Chapters 1–7) the author develops the machinery needed for his classification. If $\phi: R \to M_n(D) = S$ is a homomorphism of a ring R to matrices over a skew field D then there is an induced rank function of finitely presented R-modules, given by $\rho(M) = \text{length } (M \otimes_R S)/\text{length}(S)$, which takes values in $(1/n)\mathbb{Z}$. Abstract properties of such a function define a Sylvester rank function. The main result shows that there is a bijection between Sylvester rank functions for an algebra R and equivalence classes of homomorphisms of R to matrix rings over skew fields. Much of the earlier work concentrates on rank functions on projective modules; this culminates in a description of universal homomorphisms from hereditary rings to simple artinian rings. Another interesting development is the construction of the simple artinian coproduct of simple artinian rings.

In the second part (Chapters 8–13) of the book the author applies the methods and techniques developed in the first part to specific problems. There is much of interest here, but we mention just one item—Artin's problem. Given a pair of skew fields $F \subseteq E$ one can consider the dimensions of E as a right or left F-vector space. Artin asked whether one could have $\dim({}_F E) \neq \dim(E_F)$. P. M. Cohn constructed examples with one dimension finite and the other infinite. The author shows that for any pair of integers $a, b > 1$ there exists an extension $F \subseteq E$ with $\dim({}_F E) = a$ and $\dim(E_F) = b$.

In conclusion, this book contains new work that will be of interest to a wide spectrum of algebraists. I expect it to have influence outside of the immediate scope of the book. A careful reading is recommended and this should be rewarded by many new ideas and directions.

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