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RESEARCH ARTICLE

Countless ways to count: the functional heterogeneity of number systems

Glen Whitman

California State University, Northridge, Los Angeles, California, USA
Email: glen.whitman@gmail.com

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Abstract

Number systems are shared social technologies. They are heterogeneous, differing along dimensions of modality, base, and marking. These characteristics impart differential advantages depending on purpose and context, which explains why most cultures employ multiple number systems simultaneously. Number systems are embedded in patterns of complementarity involving both human and physical capital. Viewing number systems this way allows us to apply economic insights about production, cost, and technology to things often regarded as purely abstract cognitive conventions. Combining these insights with the literature on 4E cognition shows how concrete economic factors can shape key aspects of how humans think.

Keywords: binary; complementarity; coordination; 4E cognition; decimal; duodecimal; measurement; sexagesimal; standards

Introduction

The decimal system's contestable;
We've inches and eggs duodecimal.
Computing is binary,
Tally marks quinary,
And timekeeping near sexagesimal!
– the author

In the latter half of the 19th century, as the metric system of revolutionary France was gradually adopted by most countries in Europe, a great debate occurred in the English-speaking countries where metric had met its strongest resistance. The leading argument of metric advocates in the US and UK, then as now, was that metric's decimal character made it consistent with the decimal number system. Many of metric's vocal opponents actually agreed, but argued that existing units of measure had practical advantages, especially their multiple divisibility. As an alternative, therefore, they advocated changing the number system to match the measurement system, instead of the reverse. Some businesspeople and scholars, including Herbert Spencer (1896), supported replacing the decimal system with duodecimal (Reeve 1903, Eldridge 1903). Others, seeing the advantages of binary ratios, supported either octonary (Johnson 1891) or hexadecimal (Nystrom 1862).

What metric advocates and their anti-decimal opponents shared was a belief in consistency: one way or another, the dominant number system and measurement system should have the same base. They only disagreed on which one. Even advocates of base eight, twelve, or sixteen typically favoured reforms to assure the same base was used in *all* types of measurement.

Ultimately, neither side got its way. The UK resisted metric for nearly another century, and the US never followed. Even now, 'fully' metric countries aren't consistent about it. The UK allows various exceptions. Metric time was rejected relatively early and never made a comeback (Vera 2009). Europeans often buy milk and other beverages in half- and double-litres – an implicit concession to binary ratios. Broadening the point beyond measurement, people use a variety of number systems in daily life: duodecimal (dozens) in commerce, binary in computing, quinary in tallying scores, and sexagesimal in timekeeping. Historically, using multiple systems of numeration simultaneously was even more common. Why?

One explanation is inertia or inefficient path dependence (Arthur 1989): people don't like to change their ways, especially if nobody else does. That may indeed be part of the story. This article suggests another: that deviations from decimal may persist because they are advantageous in particular circumstances. More broadly, this paper investigates the *functional heterogeneity* of number systems – that is, how they are shaped by pragmatic needs in specific use contexts.

A common though usually implicit perspective, which I dub the *overlay fallacy*, holds that numbers are merely abstractions that describe reality without interacting with it. On this view, the only basis on which to judge number systems is their cognitive burden, i.e., how easy they are to learn and understand. This naturally favours having a single system for everything. If one system (such as decimal) is equally functional everywhere, why learn more than one?

The alternative perspective advanced here is that number systems interact with pragmatic, real-world concerns in ways that matter. Number systems are shared social technologies that arise to solve recurrent practical problems. They are embedded in concrete routines, processes, human capital, and physical capital. Their heterogeneity means they can have differential advantages, leading to a correspondence between form and function. Multiple number systems can coexist because their advantages address different demands. As economic factors alter the costs and benefits of competing production patterns, their associated number systems may change as well – albeit with a lag. One possibly surprising upshot is that economic factors, such as relative price changes, can affect our cognitive frameworks.

Beyond historical interest, the thesis has implications for how we think about planning and design. Given their apparently abstract nature, numbers look like the perfect candidate for deliberate, rational control. The planner's temptation is to design a perfectly consistent system to impose in top-down fashion on everything within its theoretical domain. But if this article's thesis is correct, the comprehensive top-down approach is misguided. States and quasi-state actors have, of course, played significant and occasionally beneficial roles in shaping number systems (especially in measurement). Yet such efforts have their limits, particularly if driven by a universalising impulse. Bottom-up processes, perhaps shaped by meso-level institutions like guilds and merchant associations, have allowed number systems to serve practical needs in specific contexts – a lesson that may apply, *mutatis mutandis*, to other institutions whose connection to practical concerns is more obvious.

The next section lays out the theoretical framework, using Harper (2010) and Whitman (2023) as starting points; a subsection covers related literature in institutional economics. The subsequent section describes a taxonomy of number systems with three key dimensions: modality, base, and marking. The section after that offers a series of historical examples to illustrate functional variation across number systems. The penultimate section connects the discussion to the 4E Cognition literature, showing how 4E can be fruitfully wedded to the economics of production and technology, enriching our understanding of how cognitive tools and economic practices coevolve. The final section summarises and concludes.

Theoretical framework

Numbers as shared cognitive-institutional technologies

Harper (2010) is, to date, the only work that addresses the role of number systems in economic behaviour and organisation, despite their obvious importance to the discipline. Numbers are usually treated as given or at least self-evident. Harper instead recognises number systems as technologies: 'objects that human beings create (oftentimes unintentionally) and that serve as an indirect means to human satisfaction. They are "tools" in the widest sense of the term' (171). Work in cognitive science has also recognised number systems as tools (e.g., Bender & Beller 2018; Kaaronen *et al.* 2024).

Conceivably, the technology of number systems could be undifferentiated: a single system governing all activities related to numeration or quantification. But, as Harper strongly implies without saying outright, actual number systems are heterogeneous. For instance, Harper distinguishes number words like 'ONE, TWO, THREE' from numerals like '1, 2, 3' (2010, 167). Elsewhere, Harper notes that Babylonian cuneiform differed from Egyptian hieroglyphics because the former was written on clay and the latter on papyrus – a difference in media that likely influenced the distinct mathematical advances made in these civilisations (2010, 181, citing Boyer 1944).

Here, I draw attention to the functionality of such heterogeneity. Although Harper gestures towards this topic (e.g., describing number sequences as having 'emergent causal properties (their "functionality")' (2010, 186), he does not develop the idea in detail. This paper seeks to fill that gap. To extend Harper's first example: number words have an advantage in speech, being immediate and audible but ephemeral, whereas written numerals offer relative permanence that can facilitate record-keeping.[1] The clay-versus-papyrus claim above implies differing advantages within the field of mathematics, but not a decisive edge for either system. This point can be broadened dramatically. As will be shown in the sections 'A taxonomy of number systems' and 'Illustrative cases of functional heterogeneity', number systems differ along multiple dimensions in ways that affect their functionality in everyday economic life.

Harper recognises the affinity between his perspective and Lachmann's work on capital (Harper 2010, 180; Lachmann 1978). Lachmann characterises capital not as an undifferentiated mass, but as heterogeneous objects embedded in *patterns of complementarity* with other productive inputs – including other forms of capital (Lachmann 1978, 35). While number systems are not capital, they are technologies instantiated in both physical and human capital. They are embedded in mechanical devices, such as calculators and measuring instruments, and accompanied by skills and behavioural patterns, such as arithmetic algorithms for making calculations and 'mnemotechnics' for producing smaller units from larger ones (Kula 1986, 85). One well-known case is the use of binary in computing, driven by its compatibility with the on/off nature of logic gates. Hexadecimal offers a cognitively accessible shorthand for binary, thus serving as a bridge between humans and computers.

Harper also connects his discussion to the growing literature on 4E (embodied, extended, enactive, and embedded) cognition – a connection I will explore in greater depth and with concrete applications. I will leave this discussion for the penultimate section of the paper, however, so that I may draw on the illustrative examples in the sections titled 'A taxonomy of number systems' and 'Illustrative cases of functional heterogeneity'.

Whitman (2023) advances a transaction cost-based theory of customary measurement systems. The present article generalises that framework to number systems more broadly, offering a typology and a cognitive-economic perspective to explain their variation and functional fit, while incorporating economic factors beyond transaction costs such as technological complementarity. Measurement constitutes a subset of numeration; specifically, the subset aimed at quantifying physical properties.[2]

Whitman argues that, at least in preindustrial times, there was good reason for customary measures not to be consistently decimal. Scarcity of standards put a premium on units with lower implementation costs, which privileged measures whose subsidiary units were easily reproducible

[1]Depending on the medium, of course; numbers written in chalk or sand will be less durable.

[2]See Wiese (2003, 40) quotation on p. 10.

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through a process of halving or doubling the base unit – leading to units with ratios such as 8:1 and 16:1. Whitman calls this the *binary principle* (2023, 713).[3] The utility of *multiple divisibility* is another contributing factor, which has sometimes resulted in 3:1 ratios that, when combined with binary, yield 12:1 ratios (Whitman 2023, 715; Gyllenbok 2018, 3). But the relative difficulty of physically implementing non-binary divisions, combined with diminishing marginal value of further possible divisions, explains why additional factors beyond a single 3 and multiple 2's are relatively uncommon in customary measurement.

Whitman introduces a refinement that will help broaden the discussion to numeration in general: the *counting principle* (2023, 715). In some quantity ranges, people rely on simply counting things that have been individually measured (barrels, bales, etc.) rather than measuring their totals directly.[4] When counting takes over, people *may* use decimal grouping – but often rely on groups of twelve or twenty instead. These groupings offer multiple divisibility without the difficulty of accurate division, since the parts are already discrete. In addition, twelve and twenty offer advantages of spatial configuration: they can be arranged in recognisable blocks like 3x4 and 4x5, which 'may also fit better in spaces such as storage rooms, carts, and cargo holds' (Whitman 2023, 716; see also Eldridge 1903). In the larger domain of numeration, these advantages will loom large in explaining non-decimal systems.

Aside from minimising implementation costs, customary measures had to solve coordination problems arising from the availability of multiple systems (Whitman 2023, 712). Like most standards, they have the potential for multiple equilibria. Coordinating on shared units facilitates commerce across different regions and trades. In some cases, those gains may drive convergence on a single system. In other cases, however, people find it cost-effective to continue using different – often trade-specific – standards, or else to effect a *partial* merger into a hybrid system (Whitman 2023, 717). As we will see, the same insight holds for number systems in general. As Harper notes, numbers are devices for 'tackling recurrent coordination problems' (2010, 171). There is no reason to assume the *same* number system will serve best in all possible domains.

Although number systems are not all created equal, some are highly substitutable. This is the overlay fallacy's kernel of truth: any two sufficiently complete number systems can, in principle, describe the same things and perform the same operations. Consequently, a slight functional advantage of System A over System B will not necessarily result in System A's dominance – a variety of path dependence (Arthur 1989, David & Greenstein 1990). However, Liebowitz and Margolis (1995) emphasise that not all path dependence is inefficient, since transition costs are real and existing systems may be supported by durable physical and human capital. Choi (2008) extends the discussion by highlighting political and structural barriers to change. A new standard's advantages must outweigh transition costs and structural impediments to justify switching. Whether 'the best' number system will prevail in any given context depends, therefore, on the strength of selection pressure.[5]

Conceiving of number systems as shared social technologies enables us to import existing economic insights about production, cost, and technology to understand what are often understood as purely abstract cognitive conventions. Numerical technologies are instantiated in production patterns whose total costs are determined by the (sometimes implicit) prices of their inputs, and must therefore compete on the basis of efficacy and cost. The most effective numerical technology may vary by activity. As relative prices and other economic factors change over time, production patterns can be expected to change – along with the number systems embedded in them.

However, new technologies do not always prevail, even over long time scales, if their advantages are not decisive. Consider a different competition between technologies: electric vehicles (EVs) versus

[3]In Whitman (2023), I used the Eye of Horus fractions (1/2 through 1/64) as a leading example of this principle. Since then, I've learned these fractions were associated with *hieratic* symbols, and their association with the Eye of Horus *hieroglyph* is likely apocryphal (Ritter 2003). However, the sequence itself is authentic and still exemplifies the binary principle.

[4]Depending on one's language, bundling things into discrete packages may transform mass nouns ('hay', 'wheat') into count nouns ('bales', 'bushels'). I thank a referee for this observation.

[5]See Chrisomalis (2020, xiv-v) for a similar perspective.

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internal combustion engines. While EVs are superior in many respects, such as reducing emissions and noise, they face clear disadvantages, especially limited range and long recharge times. Moreover, the network of EV charging stations is still nascent (at least in the US in 2025), which complicates long-distance travel. Many drivers will wait to adopt EVs until the network is more extensive – a variety of coordination problem. Consequently, combustion engines have a relative advantage in long road trips, while EVs suit everyday commuting. Cars are also durable, meaning combustion engines would likely remain on the roads for a substantial period even if EVs' advantage among new cars were undisputed.

Considerations like these are familiar when comparing vehicles and other capital goods based on differing technologies. The thesis of this article is that number systems, as cognitive-institutional technologies, ought to be treated analogously. We should expect them to be responsive to cost, efficacy, and coordination concerns.

Related literature in institutional economics

The previous section covered the most relevant topic-specific literature; here, I briefly situate the discussion within broader schools of thought.

This article's approach is broadly consistent with the New Institutional Economics (NIE) framework; see Coase (1937, 1960), North (1981), Nelson and Winter (1982), and Williamson (1985) among many others. A persistent NIE theme is showing how seemingly arbitrary institutions were often functional or efficient under specific conditions, and this applies to numbers as well.

This article's treatment of number systems as solutions to recurring economic concerns – including coordination problems – echoes Nelson and Sampat's (2001) characterization of institutions as social technologies and Denzau and North's (1994) treatment of institutions as shared mental models. It also resonates with more recent post-Northian approaches that integrate cognition into institutional analysis (e.g., Petracca & Gallagher 2020; Frolov 2023), a connection explored further in this article's section on 4E cognition. The overall perspective aligns with the ecological rationality literature, which emphasises the adaptive fit between cognitive strategies and their environments (Todd & Gigerenzer 2012).

Finally, the theory dovetails with Langlois's (2006) and Baldwin and Clark's (2002) view of standards and modularity, which treats measurement as a special coordination module for agents otherwise acting (largely) independently. This insight extends to number systems generally. Number systems provide modular interfaces for negotiations involving quantities, thus contributing to the partial decomposability of complex systems that Simon (1962) argues makes them manageable. Yet such interfaces must be shared, creating potential for coordination problems like those discussed earlier.

A taxonomy of number systems

This section explicates the diversity of number systems along multiple dimensions. But first, what *is* a number system? Harper (2010, 171, 174), drawing on Wiese (2003) and Searle (2005), posits three criteria for an object to qualify as a 'number sequence': it must be (1) a collection of well-distinguished elements, (2) constituting a progression, (3) used by a network of agents to accomplish quantitative functions. Quantitative functions include assessing 'properties of objects as diverse as cardinality, weight, temperature, rank, and identity' (Wiese 2003, 40).

Three caveats are needed. First, Harper limits the scope of his analysis to the natural numbers (2010, 171), but nothing in these criteria rules out other numerical sets: negative numbers, fractions, and so on.

Second, these criteria define what constitutes a number *sequence*, but they do not exhaust the characteristics of the number *systems* built on them. Their other features are typically what distinguish them.

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Third, seeing number systems as technologies raises the possibility of *substitute* technologies – objects that fail criterion (1) or (2) but nevertheless serve quantitative functions. For example, tally marks in the form of notches on sticks arguably fail criterion (1), as the notches are not well distinguished.[6] Nevertheless, they serve quantitative functions (especially tracking cardinality) and thus constitute a viable substitute for a full-fledged number system. Seemingly 'primitive' substitutes may persist even when more 'advanced' technologies are available.

With these caveats in mind, we may turn to the dimensions of number systems. Other typologies, such as those analysed by Widom & Schlimm (2012), emphasise characteristics like dimensionality, organisation, and extendibility that are important for their purposes. Here, I focus on three dimensions that seem most relevant to economic analysis:

Modality. Modality refers to the physical means by which elements are expressed (Chrisomalis 2020, 11). Two modalities mentioned earlier are verbal number words and written numerals. The former may be subdivided between spoken words and their written representations.[7] To these we can add the *somatic* modality: using the body to convey numbers. Examples include finger-counting and the practice in some Papua New Guinean cultures of counting by pointing to body parts in sequence (Saxe and Esmonde 2004, 12).[8] Lastly, there is the *artefactual* modality: representation of numbers in physical form on external devices.[9] Examples include abacuses, counting boards, and Incan quipus. For people who used these devices historically, the abstract idea of number could be inseparable from its physical representation. As Netz says of the Ancient Greek reliance on counters, 'We imagine numbers as an entity seen on the page; the Greeks imagined them as an entity grasped between the thumb and the finger' (2002, 329).

The distinctions between modalities are not always sharp. Inasmuch as speaking involves the mouth, the spoken modality could be considered somatic; insofar as the body qualifies as an 'external device', the somatic modality could be subsumed within the artefactual. Nevertheless, these distinctions can matter for practical and economic reasons. Verbal and somatic modalities are portable and require no manufacturing, whereas artefactual modalities' relative permanence often facilitates record-keeping. For instance, the Incan quipu allowed a form of 'material tracking and tracing … useful for the coordinating of wealth and labour at scale and at a distance' (Souleles 2020, 4).

Base. In modern mathematics, a base is a number raised to successive powers in a positional place-value system. But positional place-value is not required, and most historical number systems lacked it (Widom & Schlimm 2012, 192-194, Figure 19).[10] As I use the term here, *base* refers to how smaller units are combined into larger groups.[11] The modern Hindu-Arabic system is fully decimal, i.e., consistently based on groups of ten. Many measurements are partially duodecimal – for example, twelve inches to the foot, twelve hours to the half-day. Vigesimal (base-20) systems have appeared in various cultures, including the ancient Mayans, and a vigesimal aspect can also be seen in the once-common grouping of discrete items like fish and nails by the 'score' (Stevenson 1890, 322).

[6]We could say a set of notches constitutes the numeral for the notches' cardinality; i.e., the sequence is: (i) a stick with one notch, (ii) a stick with two notches, (iii) a stick with three notches, etc. But this seems a stretch beyond the intent of Harper's criterion (1).

[7]In logographic systems, such as Chinese, there may be no difference between the written-verbal and written-numeral modalities (Chrisomalis 2020, 156).

[8]This system violates Harper's criterion (1) because at least one body location is used for two different numbers (Saxe and Esmonde 2004, 12). Bender & Beller (2012) document the diversity of somatic number systems.

[9]These modality terms are my own, but others have classified modalities similarly, e.g., Bender & Beller's (2018) 'material, body-based, verbal, and notational' modalities.

[10]Cumulative-additive systems, for example, have distinct characters for higher-order values and repeat them as needed; Roman numerals are a well-known example (Chrisomalis 2020, 12).

[11]This broader concept of base is common in metrology (e.g., Gyllenbok 2018, 5), though less so in the study of number systems generally. Still, it aligns with Bender & Beller's (2018) dimensional approach, which allows for *irregularity* in the sense that the same base and/or subbase need not be 'applied at every power level and … used to generate all power numerals in a consistent manner' (301).

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Sexagesimal (base-60) was used by the Babylonians, and aspects of it appear in modern time measurement.

Not every number system has a single base. They may contain multiple bases – sometimes coequal, other times primary and auxiliary.[12] The notion that a number system must have a single base is probably recent. Ancient people were fairly 'promiscuous' in their groupings, using different bases as suited their needs. This could prove useful, as multiple bases allow divisibility by the factors of all bases. For example, the old British monetary system's pound consisted of twenty shillings of twelve pence each, making it divisible by all factors of twelve and twenty. Systems like this are called 'mixed-radix' and are especially common in measurement (Knuth 1998, ch. 4.1).

Why have bases at all? Simply put, they help human brains grasp large quantities by breaking them into recognisable groups. Larger groups can be treated as single units; a merchant can count dozens of eggs in much the same way he counts individual eggs. Thus, a base structure makes a number system decomposable (Simon 1962), even as the system itself functions as a module within the broader realm of economic activity.

We should also be cognizant of how decimal has been superimposed on other number systems, especially in measurement. As a result, a system may appear decimal in its symbols yet have a fundamentally non-decimal character. Time is an excellent example. When we say something is scheduled for 12:45 p.m., the '12' and '45' are rendered in decimal numerals. Yet the grouping system reflects a mix of duodecimal and sexagesimal.

Marking. The third dimension is *marking*, meaning elements that provide additional context or information about the function being performed. The simplest examples occur with written numerals, where punctuation does the heavy lifting. In the US, people know 5'11" is a height, 5:11 a time, 5/11 a date, and $5.11 a price because of marking. In speech, markers may be morphemes such as the *-st* and *-th* suffixes used in English to indicate ordinality. In the artefactual modality, the thread colours in quipus could indicate the commodities being quantified (Mackey 2002, 503-4). In the somatic modality, the presentation of a gesture might distinguish a buy from a sell offer – as with the hand signals used on commodity trading floors (Garner 2010, 45).

In calling modality, base, and marking *dimensions*, I mean to convey that they are cross-cutting. We might imagine a number system being 'designed' by selecting a modality from column A, a base (or bases) from column B, and one or more markings from column C. For instance, the somatic modality might combine with a duodecimal base through the practice in some cultures of using the thumb to count the three segments of the other four fingers (Huylebrouck 2019, 51); marking could be added by using different hand presentations to indicate an offer or acceptance.

Illustrative cases of functional heterogeneity

In this section, I present several historical cases selected to illustrate the functional heterogeneity of number systems – specifically, how differences along the above dimensions can correspond to practical needs.

Tally sticks

As mentioned earlier, tally sticks constitute a 'primitive' form of counting that doesn't satisfy the formal requirements for a number sequence. Cultures worldwide have used some form of tally sticks (Baxter 1989). Although not suited to all purposes, tally sticks were remarkably serviceable for recording cardinality. For example, shepherds could rely on a one-to-one correspondence between notches and sheep to ensure the whole flock came home each night.

[12]See the sections 'Roman fractions', 'Duodecimal counting in Europe', and 'Sexagesimal counting in Sumer' for examples.

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In the European 'double tally stick' system, trading partners marked transactions – such as items bought on credit – on a single stick that was then split down the middle. Each party kept one half as a record that could be verified against the other; neither party could add or subtract a mark without causing a mismatch (Baxter 1989, 50). Versions of this system lasted well beyond the Middle Ages (ibid.), despite the availability of ostensibly superior alternatives such as Roman and Hindu-Arabic numerals.

The special advantage of tally sticks was combining easy adjustment (new notches could be added easily and sliced off when necessary) with a durable record to facilitate monitoring or contract enforcement. The double tally stick presumably faded only when alternative means of recording and enforcing contracts became readily and cheaply available.

Bullae and tokens

The ancient Sumerian and Elamite *bullae*-and-tokens system offered similar advantages. A *bulla* was a hollow clay envelope that could be filled with clay tokens and sealed. The tokens represented quantities, such as goods owed or promised for delivery. The bulla's closure, along with one party's seal, made it costly or impossible to change the contents before payment or delivery. Thus, the *bullae*-and-tokens served as an administrative record (Schmandt-Besserat 2010, 32) and possibly a tool of contract enforcement (Goetzmann 2016, 25-27), though the latter is disputed. As with tally sticks, *bullae*-and-tokens' relative permanence suited them to these purposes.

Notably, the tokens came in various shapes – and some bore 'markings in the form of incised lines' – to distinguish among different commodities (Schmandt-Besserat 2010, 28), thereby providing transactional clarity in a time when non-numeric writing was still nascent. This is a straightforward example of markings serving an economic function.

Roman and mediaeval finger reckoning

We often imagine the Romans doing all counting and calculations with Roman numerals. But this was not the case. A remarkable form of finger-reckoning was used throughout the Mediterranean during the Roman Empire (Williams & Williams 1995, 590), and ample evidence suggests it was used across 'the medieval Latin West, the Byzantine East, and all of medieval Islamdom' (Richardson 2017, 173). In this system, three fingers in various positions represented units, while the index finger and thumb represented tens. The same gestures on the opposite hand could represent hundreds and thousands. Although Roman numerals lacked both place value and zero, finger reckoning implicitly had both: place value by reserving specific fingers for specific orders of magnitude (Maher & Makowski 2001, 377), and zero by allowing the fingers to rest in a neutral position.

What purpose did this number system, operating in parallel with Roman numerals, serve? Although it is unknown exactly how its users performed calculations, there is good evidence they did so (Williams & Williams 1995, 595). Having zero and place value would have suited it to algorithmic calculations analogous to modern pencil-and-paper methods. This likely made finger reckoning useful in marketplace settings when quick math was needed or an abacus was unavailable (Turner 1951, 69).

But other explanations have been suggested. Some say that it eased trade between speakers of different languages, though this has been challenged (Williams & Williams 1995). Others claim that gestures helped reinforce understanding in noisy marketplaces (Williams & Williams 1995, 594). Finally, because gestures can be touched, they may have enabled secrecy in bargaining, as shown by 'hidden tactile negotiation' in present-day Somaliland (Musa & Schwere 2019) and historically in parts of India and the Middle East (Bayley 1883, 4).

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Roman fractions

Roman numerals have a quinary-decimal structure: a primary base of ten (as shown by X, C, and M) with an auxiliary base of five (V, L, and D).[13] In the fractional realm, however, the Romans went duodecimal. The unit was divided into twelve *unciae*. Each *uncia* was subdivided into 24 *scrupuli* – essentially, another (negative) power of twelve, halved once more (Wyatt 1964, 269). Tellingly, Latin had a distinct name for every *uncia* from 1/12 to 11/12, but *not* for every tenth from 1/10 to 9/10 (Glare 1968). Some Roman abaci had a column for *unciae*, with six treated analogously to five in the whole-number columns: a single bead above the line (Wyatt 1964, 271n6).

Why were Roman fractions duodecimal? After all, the Romans clearly had access to a decimal way of thinking. But the realm of fractions is where twelve's superior divisibility is most advantageous. This explanation is supported by the system's origin in weight and land measurement (Maher & Makowski 2001, 378). Although it originally applied to fractions of the *as*, a Roman currency and weight unit, the system was generalised to other domains – including *downward* division of larger things counted *upward* decimally. The term *as* came to refer not only to currency but to any 'whole', such as an estate (Stewart 2019, 98). A treatise by the Roman jurist Maecianus 'affirms the primacy of the duodecimal system in Roman fractional calculations of the time, and ties this directly to the dividing up of money and property' (ibid.).

Duodecimal counting in Europe

When it comes to counted measurements, dozens were common historically in Europe, and to some extent in modern times. Sometimes they are even raised to higher powers: a *gross* is twelve dozen, and a *great gross* is twelve gross (Darling 2004, 140). The practice was widespread; as Kula notes, 'As far as transactions *involving counting* are concerned, it would appear that the duodecimal system prevails throughout Europe: the dozen rules, assisted by its divisions and multiples' (1986, 83, emphasis added). The word 'counting' is key here, distinguishing these uses from direct measurement, where binary divisions were more common (ibid., 85). In short, twelve's multiple divisibility privileges it in a specific context – counting discrete items – where the practical difficulty of accurate division is absent.

Aside from powers of twelve, preindustrial European commerce often employed the counterintuitive 'long hundred' of 120 and the 'long thousand' of 1200 (Ulff-Møller 1991). This hybrid system combined two bases, ten and twelve, and was so familiar that the Roman numeral 'C' could denote either 100 or 120 (Ulff-Møller 1991, 327).

How might this system have emerged? Multiple divisibility is again the likely suspect. A long hundred could be constructed as *either* ten groups of twelve *or* twelve groups of ten. Likewise, the long thousand could be ten long hundreds *or* twelve short (decimal) hundreds. Ulff-Møller surmises that these constructions were typically used for different purposes: groups of twelve (or 120) when counting upward, groups of ten (or 100) when dividing downward (1991, 327). The latter method leverages the divisibility of 12: a third of a long hundred is forty; a quarter of a long thousand is three (short) hundreds. This pattern parallels the Roman use of duodecimal fractions in the downward direction.

Although divisibility is part of the long hundred's story, there is a link to coordination problems. Some evidence suggests a ten-twelve-hybrid system was used by Teutonic people before Christianization, with the long hundred being the *original* hundred (Stevenson 1890, 313). Later, the encounter with Roman numerals and Christianity introduced the decimal hundred (ibid., 317), after which the two hundreds coexisted; as Ulff-Møller puts it, 'the long and the short hundred interact' (1991, 325). This suggests that both were useful – for the reasons above – but *also* that the long-hundred system effected a compromise between different coordinative equilibria, accommodating people accustomed to working with tens *and* people accustomed to working with twelves.

[13]Five constitutes an auxiliary base (or subbase) because there is no special role for powers of five such as 25 and 125.

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The morphemes 'long' and 'short' (or their equivalents) served as verbal markings to distinguish the two hundreds – but such indicators were often missing, leaving readers to infer the meaning from context (ibid., 325). This lack of marking may have inhibited the system's functionality.

Duodecimal systems have appeared in other cultures, including some in the Plateau of Nigeria (Hammarström 2010, 28). Still, Europeans seem to have had a special affinity for dozens; examining the reasons would merit a longer treatment.

Sexagesimal counting in Sumer

Many have noted the similarities between the Sumerian sexagesimal system and European long hundred, with some even positing a historical influence. That influence now seems unlikely (Ulff-Møller 1991, 323). But the striking similarities suggest a similar story about the Sumerian system's functionality.

The origins of Sumerian sexagesimal remain unknown, but several hypotheses have been offered: that it arose from an encounter between base-6 and base-10 cultures (Thureau-Dangin 1939, 98); from multiplying the twelve finger segments of one hand by the five fingers of the other (Huylebrouck 2019, 51); from an encounter between base-10 and base-12 cultures, with 60 as the least common multiple (LCM) (Ifrah 1981/1998, 93); from the need to relate two distinct units of measure, one substantially larger than the other (Neugebauer 1927, 44-45); or from efforts to rationalise multiple units of measure with various ratios (Rahmstorf 2010, 101–102).

The first hypothesis is supported by the form of Sumerian written numerals, which are visibly decimal from 1 to 59, consisting of clusters of 'tens' and 'ones'. In terms of overall structure, Sumerian numbers are a mixed-radix system. Higher orders of magnitude are formed by alternating between 10 and 6: first multiply by 10 to get 10, then by 6 to get 60, then by 10 to get 600, then by 6 to get 3600, and so on (Thureau-Dangin 1939, 104).[14] The 6-meets-10 hypothesis is further supported by archaeological evidence of a base-6 civilisation that may have migrated into Mesopotamia (Laki 1969). All the above hypotheses can find some support in Sumerian units of measure, which exhibit ratios of 6, 10, 12, 20, and, of course, 60 (see Gyllenbok 2018, 565).

I am not qualified to adjudicate among these hypotheses. But notice that most share something in common: an encounter between, and merger of, different systems. The competing systems may have come from different cultures. Or they might have come from different trades, with each using the groupings best suited to their differing needs in production, packing, distribution, and sale. In that situation, 60 might have become a coordinative focal point (Schelling 1960, 57) because it could be reached by counting in tens, twelves, or twenties. When traders using dozens met traders using tens, for example, the LCM would have been a natural stopping place ('I will trade my five dozen for your six lots of ten'). This is not necessarily an origin story; it might instead describe the system's persistence, however it emerged.[15] Either way, it shows how number systems can reflect a blend of practical needs and coordinative concerns.

Timekeeping

Although sometimes attributed to Babylon, the 24-hour day's actual origin lies in ancient Egypt (Neugebauer 1969, 81-82). Daytime and nighttime were sharply distinguished and allotted twelve hours each. Although these hours – especially at night – were mapped onto the apparent movement of

[14] As Thureau-Dangin observes, the Sumerian system appears simply sexagesimal only if one ignores every other order of magnitude. Widom & Schlimm classify it as base-60 with subbase 10 (2012, 194 Fig. 19) and implicitly as Type 3 in their subbase typology (158–159, Fig. 1). That classification, while mathematically equivalent to Sumerian, does not fully capture the system's mixed-radix character. The later Babylonian system eliminated most of the mixed-radix elements and is more comfortably characterised as base-60 subbase-10.

[15] Ifrah characterises this LCM origin as a 'learned' solution that requires 'too much intellectual sophistication' to be plausible (1981/1998, 93). I disagree because of the focal-point explanation just offered.

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the stars (ibid.), the stars' positions do not inherently favour twelve over (say) ten or sixteen. We may surmise that twelve benefited from its usual advantage of multiple divisibility. Indeed, dividing the workday and corresponding day-wage into halves, quarters, and thirds was common in the Middle Ages (Dohrn-van Rossum 1996, 311). Fully binary divisions were less useful because periods of time – unlike lengths of cloth or volumes of liquid – could not be directly compared to each other. Hours were therefore always going to be approximate, and their length varied seasonally, a practice that persisted for centuries (Landes 2000, 438n18). In this context, divisibility trumped precision.

The 60-second minute and 60-minute hour are also often attributed to Mesopotamia. But the Sumerian divisions of the day bore little resemblance to ours, with sexagesimal playing a negligible role.[16] Later scholars inherited the 360-degree circle, with its 60-part subdivision of degrees, from the Babylonians (Merzbach & Boyer 2011, 152). While these divisions mattered in astronomy and geometry, they played no role in everyday timekeeping (Dohrn-van Rossum 1996, 282). Even among scholars, sexagesimal time divisions differed greatly from modern ones. Ptolemy divided the day into 360 *moîrai*, equal to four modern-day minutes; in later centuries, the *minutum* could refer to '1/15 hour (4 min.), 1/10 hour (6 min.), and 1/60 day (24 min.); but it never denoted 1/60 hour, which was an *ostentum*' (Holford-Strevens 2005, 9). Scholars adopted the 60-part time divisions in the 'later Middle Ages' (ibid.), and common people in Europe did not regard the hour as having 60 minutes until the late 16th century (Dohrn-van Rossum 1996, 282), by which time clockmakers had begun adding minute hands (Cipolla 1978, 50).

The Mesopotamian inheritance was not, therefore, a continuous timekeeping tradition maintained out of inertia or path dependence; European clockmakers *chose* to adopt sexagesimal. Why? They were likely influenced by scholars' use of base-60, and also fascinated by ancient civilisations (Englund 1988, 122). But there was a more practical reason. In everyday European life, the hour 'was divided into halves, thirds, quarters, sometimes into twelve parts' (Dohrn-van Rossum 1996, 282). When finer divisions became desirable, they needed to align with the conventional fractions people already found useful. A 60-part division fit the bill, while also allowing division by 5 and 10. Furthermore, it could be easily superimposed on a 12-hour dial, while a 100-minute division could not.

Roman versus Hindu-Arabic numerals in Europe

Hindu-Arabic numerals were introduced to Europe as early as the 10th century and became more widely known when Fibonacci worked to popularise them in the 12th century (Chrisomalis 2020, 79, 105). Yet Roman numerals predominated over Hindu-Arabic numerals in Europe well into the 16th century (ibid., 107), including among people aware of the supposedly superior system. Was this intransigence or path dependence? Perhaps – but there is another explanation.

Although written calculations with Roman numerals are possible, most people did their calculations with a counting board, also known as the Western abacus.[17] These devices may have been supplemented by finger-reckoning for simpler sums or preserving intermediate results (Williams & Williams 1995, 588-589). There were practical reasons for these methods – most notably, the scarcity of writing materials throughout the Middle Ages (Sugden 1981, 14). Papermaking did not become widespread in Europe until the 15th century; before then, writing required costly parchment made from animal skins (Hoffmann 2024). Meanwhile, counting boards were durable capital goods, and fingers were always available. Both methods already embodied place value and zero, the primary virtues of Hindu-Arabic numerals. Furthermore, counting boards nicely complemented Roman numerals. In typical European versions, each row represented a decimal order of magnitude, with fives represented by a single token between the lines. This structure created a one-to-one correspondence between tokens

[16]The Sumerians divided the full day (not just half) into twelve *danna* of 30 *giš* each. A *giš* was equivalent to four modern minutes. Sexagesimal appears only in the division of the *giš* into 60 *ninda* (Thureau-Dangin 1939, 112-3). But it is doubtful common people ever used *ninda*, lacking any means of measuring time so precisely.

[17]A counting board is a grid with rows or columns for placing tokens – essentially an abacus without rods.

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and the quinary-decimal Roman system (Netz 2002, 327). For instance, CLXXVI (176) would be represented by six tokens, one per character – enabling an easy translation from calculation to recording the final answer.[18]

More to the point, there were really two patterns of technological complementarity: Roman numerals paired with the counting board (and sometimes finger-reckoning), and Hindu-Arabic numerals paired with pen and paper. Switching from Roman to Hindu-Arabic was not just a matter of cognitive adjustment. It was a matter of changing a whole pattern of economic behaviour. People had durable physical and human capital in the old system, and high input costs of adopting the new system. All things considered, there were good reasons to stay put until the complementary inputs for Hindu-Arabic numerals became cheaper. In Liebowitz and Margolis's (1995) terminology, this was a case of second-degree path dependence – where persistence results from pragmatic constraints and technological complementarity rather than genuine inefficiency. Eventually, as innovation drove down paper manufacturing costs and papermaking spread through Europe during the late Middle Ages and Renaissance (Harford 2017), switching became economically viable.

This story does not necessarily rule out inefficient path dependence; it is possible that Roman numerals persisted even after Hindu-Arabic became cost-effective. Chrisomalis (2020) attributes the eventual shift to the printing press and rising literacy, which along with new arithmetic texts allowed the new numerals to 'develop[] a critical mass of use alongside a critical mass of new users in the sixteenth century especially . . . until their frequency cemented their position as the dominant notation of the region' (Chrisomalis 2020, 115). This is essentially a tipping-point story, wherein an influx of newly educated people shifted the coordinative equilibrium from the old technology to the new. However, Chrisomalis also emphasises that, for some time, the two systems coexisted, often side-by-side (ibid., 108-109; Sugden 1981, 14), implying that users found both systems adequate for their purposes. While Hindu-Arabic had advantages that likely increased over time, the selective pressures were apparently not strong enough to generate a speedy shift. For most of the Middle Ages, Roman numerals were part of a production pattern that was at least not markedly inferior – and may well have been superior until paper became cheap and abundant.

Number systems and 4E cognition

Number systems provide a compelling exemplar of key ideas from the 4E cognition paradigm, whose influence in institutional economics is growing. Moreover, this article's approach suggests how 4E cognition theorists might incorporate insights from economic theory. Just as 4E cognition has informed economics, economics may, in turn, inform 4E cognition. I support this claim by showing how each 'E' relates to factors such as technology adoption, capital structure, responsiveness to relative prices, and coordination on shared standards.

4E is an umbrella term for four related perspectives – designated as *embodied*, *extended*, *embedded*, and *enactive* – that emphasise how human cognition draws on resources from outside the brain. These 'externalist' views contrast with orthodox ('internalist') views that locate cognition entirely within the brain (Newen *et al.* 2018, 4).

It's important to note that *embodied*, *extended*, *embedded*, and *enactive* are used in ways that are overlapping, sometimes conflicting, not fully distinct, and not always consistent with the vernacular meanings of the words. Most prominently, *extended* and *embedded* do not usually denote two distinct aspects of human cognition, but rather two ways of characterising the same aspects of human cognition. *Extended* usually treats the human mind as literally extending into the environment, whereas *embedded* sees the mind as distinct from but nevertheless embedded in an external environment (Rowlands 2010, loc 1282-89; Rupert 2009, 35). Some theorists do not even regard embedded cognition as a full-fledged 'E' in its own right, but rather as an argumentative foil for extended cognition. *Enactive*

[18] This correspondence was less perfect with subtractive Roman numerals, such as IX (9). However, subtractive notation was optional.

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cognition is sometimes treated as an overarching perspective that encompasses both extended and embodied cognition, while emphasising the dynamic interplay between agent and environment (Petracca & Gallagher 2020, 754n7). Yet some theorists see a tension between embodied and extended, as the former highlights how cognition is crucially shaped by the physical medium (body) involved, whereas the latter tends to abstract from the medium to focus on functional similarities (Clark 2008). The fluidity of 4E terminology makes it difficult to draw sharp distinctions, especially across disciplinary boundaries.

In this article, I use all four terms to characterise relevant aspects of how people interact with number systems. I will lean on these words' vernacular meanings in a way that treats them as distinct yet compatible. My hope is that this approach complements the 4E perspective without doing too much violence to these terms' past usage in the field. I intentionally sidestep the debates among 4E perspectives, especially the extended-versus-embedded debate. Scholars from all 4E camps should find my claims about number systems broadly compatible with their perspective, although they might prefer different language.

Embodied cognition. Embodied cognition theorists emphasise how humans use their bodies to support cognitive processes. 'Cognition is embodied when it is deeply dependent upon features of the physical body of an agent, that is, when aspects of the agent's body beyond the brain play a significant causal or physically constitutive role in cognitive processing' (Wilson & Foglia 2011). Finger counting is a common illustration of this idea (e.g., Fischer & Brugger 2011; Bender & Beller 2012). Indeed, any number system with a somatic modality likely qualifies as embodied cognition. A deeply somatic imprint appears in the modern decimal system; if humans had eight or twelve fingers, its base would likely be different as well. Yet decimal is not the only somatically influenced option. Historical vigesimal systems probably arose from counting on both fingers and toes; quinary subbases, as in Mayan and Roman numerals, reflect the importance of the single hand.

It is tempting to relegate finger-counting to the status of origin story, useful today only in teaching children. But Roman finger-reckoning shows how sophisticated embodied aids to cognition can become. It had implicit place value and zero. It served functions including calculation, secrecy, and communication in loud marketplaces. This shows how the right economic circumstances can foster greater reliance on embodied cognition over other cognitive strategies (such as mechanical devices). In this way, cognitive tools can be competing production technologies that prosper in different contexts.[19]

Extended cognition. Although *extended* often denotes the view that cognition literally takes place outside the human body, here I use it in its more contained sense: recognising that cognitive activities may rely upon 'processes and structures that occur outside the body in the wider environment' (Rowlands 2010, loc 542). These structures are typically 'extrabodily components or tools' (Newen *et al.* 2018, 6) or 'external physical devices' (Varga 2016, 2469). The connection to number systems with an artefactual modality is straightforward: tally sticks, abacuses, rulers, and scales all qualify as forms of extended cognition – or, in Zhang & Norman's (1995) terms, 'distributed numerical cognition'. Some argue that clay tokens (the kind used with bullae) *preceded* the abstract concept of number and supported its development (Malafouris 2010, 39-40).

Extended cognition theorists often emphasise *skilled use* of artefacts (Clark 2003). Such skill is obvious with abacuses, which work best in the hands of experienced practitioners. Skill also manifests in mnemotechnics, i.e., behavioural algorithms that effectively mimic cognitive processes such as counting or arithmetic.[20] As one example, Kula (1986, 83) quotes from a letter from the Commission for Weights and Measures of the Cisalpine Republic (Northern Italy) in 1800, which had encountered resistance to metrication: 'Every girl and every unlettered tailor know what half a quarter-ell stands for; but we would lay a hundred to one that many professional accountants would be unable to assure you that half a quarter-ell is equal to one hundred and twenty-five thousandths'. In a system based on

[19]This is not meant to rule out complementary cognitive strategies; see below.

[20]There is a natural connection here to ethnomathematics, which examines how different societies have developed cultural practices that achieve mathematical ends; see D'Ambrosio (1985) and Ascher (1991).

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binary divisions, workers could produce a smaller unit from a larger one by splitting it three times ('half a quarter'), without necessarily understanding the underlying math. Skilled artefact use illustrates how different inputs (physical capital and human capital) can be tight complements in production.

Enactive cognition. Enactive cognition theorists emphasise that cognition 'involves an active engagement in and with an agent's environment' (Newen *et al.* 2018, 6). In the present context, enaction means that number systems do not typically arise in the abstract, but from recurrent problem situations – specifically, those involving a need for quantification, such as keeping track of livestock, dividing goods among parties, settling terms of trade, and maintaining records of debts. We are talking again about mnemotechnics, but now focusing on *why* they arise: to solve practical problems in everyday life. Successful methods tend to be copied; as a result, useful knowledge becomes embedded in practice.[21] People may then incorporate these practices into their basic view of the world; recall how Roman weight measurement gave birth to a general system for all fractions. To the Romans, fractions simply *were* duodecimal. Their cognition was shaped, ultimately, by practical concerns.

One especially important kind of recurring problem situation is coordination. The challenge is not just to quantify, but to do so in a way that others can understand and verify. Number systems, as well as the devices and techniques for manipulating them, therefore tend to be communal. While in principle an individual could adopt an alternative method, the structure of a coordination game creates incentives to conform (or else risk exclusion from communication and trade). As people internalise shared methods, the equilibria of coordination games become routine parts of their cognitive apparatus. Number systems thus qualify as 'economic cognitive institutions' in the sense of Petracca & Gallagher (2020).[22] However, sufficient changes in economic circumstances can prompt some people to adopt alternative conventions; if enough others follow, the system may shift to a new equilibrium. This is consistent with the enactive view's emphasis on the two-way interaction between individuals and their environment.

Embedded cognition. Embedded cognition theorists emphasise that humans 'structure our environments to facilitate problem-solving; we create scaffolding on which human cognition depends' (Rupert 2009, 207). This environment is explicitly social, with scaffolding provided by 'technological and cultural creations' (ibid., 151), including prevailing equilibria in coordination games. Although embedded cognition is often treated as the less-radical cousin of extended or enactive cognition, in my usage, it is arguably more radical. It means that systems of numeration are never chosen in a vacuum, but are instead selected to fit into broader environments that include both scarcity of resources and other institutions. Moreover, number systems come in the relatively stable packages that I've called patterns of complementarity. Complementarity matters because the cost of any input can hoist or hinder all other elements in the same pattern. The best example is Roman versus Hindu-Arabic numerals. The former came in a package with the abacus or counting board plus finger-reckoning, the latter in a package with writing instruments and paper. Scarcity of paper could thus influence the acceptance or rejection of an entire package, including its embedded number system.[23]

As in Lachmann's discussion of substitutes and complements, alternative number systems are substitutes – but at the level of the pattern rather than as an isolated practice. You cannot simply swap in a new technology while everything else remains the same. To substitute is to adopt a different package of complementary inputs (Lachmann 1978, 56-57). As D'Adderio (2011) puts it, 'configurations of artifacts and people are stabilized in recurrent – but continuously challenged – patterns of interaction' (2011, 199). The relative costs of competing complementarity patterns can explain the maintenance of an old pattern; changes in relative costs can explain switching to a new one.

[21] Compare Hodgson's (1988, 126-127) treatment of habits as devices for 'tacit knowing' and acquisition of skills.

[22] Petracca & Gallagher (2020) frame their theory of economic cognitive institutions in terms of extended cognition, but explicitly ground it in an enactivist framework (754n7).

[23] Compare Choi's (2008, 199) claim that Korea's switch from Chinese characters to a phonetic alphabet was aided by the arrival of typewriters.

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Hence, we may be able to explain some aspects of our cognition by reference to the relative prices of their complementary inputs.

This last insight suggests a promising direction for research at the intersection of cognitive science and economics: to see whether other cognitive institutions are similarly responsive to economic incentives such as implicit prices.

Conclusions

It is common to think of number systems as belonging to the rarefied and abstract realm of pure mathematics. To the contrary, I have argued that they arose historically for practical purposes, and those purposes shaped their forms. Number systems are heterogeneous, differing along dimensions of base, modality, and marking. Because these features offer differential advantages depending on context and purpose, we should not expect a single number system to be used for everything. Historically and even today, we see different systems used alongside each other – and this heterogeneity is practical.

Future research might explore how broad economic patterns – such as dominant occupations, modes of production, and scarcity of materials – have shaped different societies' numerical practices. While this remains hypothetical for now, a suggestive parallel can be seen in how different trades within a single society have historically developed distinct units of measure (Whitman 2023, 724).

Another direction concerns the symbolic and institutional authority of number systems. Work in the quantification literature (e.g., Porter 1995, Gooday 2004) emphasises how numbers can legitimise decisions and project rationality in bureaucratic and scientific contexts. While these dynamics are typically associated with how numbers are used rather than how number systems are structured, they too may have influenced the adoption or design of number systems – particularly in periods of state-led standardisation.

Once we conceive of number systems as shared social technologies, we can see how they are shaped by the same economic forces that drive other technologies. One such force is the emergence of coordinative equilibria to accommodate competing commercial needs. Another is patterns of technological complementarity, whose value can be boosted or burdened by price changes of any component. Yet another is the persistence of 'primitive' substitutes that outperform full-fledged number systems in specific contexts. We can see these forces at work in a range of historical examples. By merging these insights with 4E cognition, we find that economic forces can shape not just the world around us, but some basic aspects of how we think and understand.

Further, the correspondence between number systems and the specific problems they arise to solve lends support to a broader proposition: that bottom-up processes can produce effective and functional economic institutions. This is not to deny the role of top-down interventions such as legal mandates, educational reforms, and standardisation efforts. States, empires, and other institutional players have exerted influence over numeration systems for political, commercial, and administrative purposes. For such efforts to be lasting and effective, however, they must align with pragmatic needs and deeply embedded behaviour patterns. While number systems may be subject to planning and reform, their long-term viability depends on reflecting the decentralised bottom-up processes that helped shape them.

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