J. Fluid Mech. (2025), *vol.* 1022, F1, doi:10.1017/jfm.2025.10730

JFM FOCUS ON FLUIDS

Quantum turbulent flows: a model for classical turbulence?

Luca Galantucci

Istituto per le Applicazioni del Calcolo 'Mauro Picone', Consiglio Nazionale delle Ricerche, via dei Taurini 19, 00185 Roma, Italy

Corresponding author: Luca Galantucci, luca.galantucci@cnr.it

(Received 9 September 2025; revised 9 September 2025; accepted 15 September 2025)

Quantum turbulence is characterised by the collective motion of mutually interplaying thin and discrete vortex filaments of fixed circulation which move in two mutually interacting fluid components. Despite this very peculiar nature determined by quantum-mechanical effects, turbulence in quantum fluids may exhibit very similar features to classical turbulence in terms of the vortex dynamics, energy spectrum and decay and intermittency. The recent work by Blaha *et al.* (2025 *J. Fluid. Mech.* 1015, A57) reveals an additional classical behaviour of quantum turbulence, by showing that the trajectories of starting vortices shed by accelerating airfoils in a quantum fluid are almost indistinguishable from their counterpart in classical viscous flows. These results strongly support the suggestive idea that turbulent flows, both classical and quantum, may be described by the collective dynamics of interacting, thin and discrete filaments of fixed circulation.

Key words: quantum fluids, turbulent flows, vortex dynamics

1. Introduction

Turbulence is ubiquitous in the universe. It occurs in systems ranging in size from tens of micro-metres, such as cryogenic helium channels and clouds of few thousands atoms confined by optical lasers, to planetary scales, such as large-scale atmospheric and oceanic flows, to astrophysical scales characterising nebulae of interstellar gas. The dynamics of turbulent flows is shaped by the mutual interplay of nonlinearly interacting vortical structures spanning a wide range of scales. Properties of these vortices may differ significantly depending on the nature of the fluid. In classical viscous fluids, vortices are unconstrained: classical vortices, such as a teaspoon-stirred vortex in a cup of coffee or an atmospheric tornado, may possess any shape, orientation, size and strength, the latter quantified by the circulation of the fluid's velocity field around a closed loop surrounding the vortex structure.

Very dissimilar properties instead characterise vortices in quantum fluids. The latter are fluids which, due to Bose–Einstein condensation of the constituent particles, exhibit quantum-mechanical properties at a macroscopic level. Examples of such fluids are liquid phases of He^3 and He^4 (Barenghi, Skrbek & Sreenivasan 2023), ultracold atomic gases (White, Anderson & Bagnato 2014), polariton condensates (Panico *et al.* 2023) and the interior of neutron stars (Haskell & Melatos 2015). In the present work we will mainly focus on the low temperature liquid phase of He^4, commonly referred to as helium II, which can be described as an intimate mixture of two inseparable fluid components (Landau 1941): the superfluid component, related to the Bose–Einstein condensate, possessing vanishing entropy and viscosity, hence similar to an inviscid, classical Euler fluid; the normal fluid, almost akin to a classical viscous fluid, carrying the whole entropy of helium II. Two density and velocity fields characterise helium II, where two motions can therefore occur simultaneously at the same position in space. Whereas the total density is almost constant in the temperature range where the helium II phase exists, $0 \leq T \leq T_\lambda = 2.17$ K (T_λ being the transition temperature), helium II is almost entirely superfluid below 1.3 K and fully normal at T_λ. Above T_λ, the liquid phase of He^4 is a classical Newtonian fluid. Most importantly, quantum-mechanical restrictions confine the whole vorticity of the superfluid to effectively one-dimensional vortex filaments of quantised circulation $\kappa = h/m$, where h is the Planck constant and m the mass of a helium atom. These quantum vortices interact collectively with each other via the Biot–Savart law, may reconnect (Bewley *et al.* 2008; Serafini *et al.* 2017) as a result of small-scale quantum compressible effects at the vortex core level and exert a so-called mutual friction dissipative force on the normal fluid (Hall & Vinen 1956). In brief, the motion of superfluid vortices is influenced by the normal fluid whose dynamics, in turn, is also governed by the mutual friction force exerted by the vortices: the overall motion of helium II is the result of two-phase, fully coupled dynamics (Galantucci *et al.* 2020).

Despite the strongly dissimilar nature of classical and quantum fluids just described, under certain conditions, turbulence in such fluids may surprisingly exhibit very similar features. The work by Blaha, Xu & La Manita (2025) precisely reports evidence in this direction, showing that starting vortices shed by airfoils accelerating uniformly from rest behave very similarly in helium II and classical Newtonian fluids. In particular, the trajectories of quantum vortices determined experimentally are almost indistinguishable from the trajectories of vortices shed in a classical fluid computed via a direct numerical simulation of the Navier–Stokes equations. In addition, a recent study by the same group (Švančara & La Mantia 2022) shows that macroscopic vortex rings in helium II propagate in a similar manner to turbulent vortex rings in a Newtonian fluid, as confirmed numerically (Galantucci *et al.* 2023*a*). The significant relevance of the investigation by Blaha *et al.* (2025) is hence its contribution in constructing a detailed map identifying the parameter range (temperatures, flow length scales, flow generation methods) where quantum turbulence (QT) in helium II behaves classically. The construction of this map is of fundamental importance not only to enhance our understanding of why turbulence in quantum fluids shows similarities to turbulence in classical viscous fluids. The extremely small kinematic viscosity of helium II (up to three orders of magnitude smaller than that of air) would indeed also allow the investigation of high Reynolds number flows of classical Newtonian fluids in helium II experimental facilities at significantly smaller scales than found in typical wind tunnels. Finally, as we shall briefly describe in the next section, the similarities between QT and classical turbulence (CT), strongly support the conjecture that QT can be considered the skeleton of classical flows: the study of the dynamics of a collection of discrete filaments of fixed circulation could enhance our understanding of the still open issues in CT.

2. Quasi-classical quantum turbulence

The characteristic presence of a tangle of quantised vortices in turbulent flows of helium II immediately introduces an additional length scale to helium II systems, the average spacing between vortices $\ell = 1/\sqrt{L}$, where L is the vortex line density defined as the average vortex length per unit volume. The current understanding is that, when turbulence is driven mechanically at scales larger than the average intervortex spacing ℓ, three-dimensional QT exhibits a classical behaviour in terms of energy decay, the energy spectrum, the statistical distribution of velocity and acceleration components and circulation statistics.

In QT, the decay of the turbulent kinetic $\mathcal{E}(t)$ is investigated in terms of the decay of the vortex density $L(t)$ assuming, as in a rotating bucket (Peretti *et al.* 2023), that the superfluid vorticity ω_s is related to L by the expression $\omega_s = \kappa L$. In this framework, the classical energy decay rate $\mathcal{E}(t) \sim t^{-2}$ (Touil, Bertoglio & Shao 2002) maps in QT to $L(t) \sim t^{-3/2}$. Measurements in helium II indeed observe this vortex line density decay rate, in distinct experimental settings (Skrbek & Sreenivasan 2012): towed-grid experiments, impulsive spin down of cubic shaped containers and intense ion injection.

The other remarkable similarity between mechanically driven QT at large scales concerns the energy spectrum $E(k)$, where k is the wavenumber magnitude. Several experiments have in fact reported that, in the inertial range $k_I < k < k_\ell$, where k_I is the inverse of the energy injection scale and $k_\ell = 2\pi/\ell$, the kinetic energy spectrum of helium II displays the celebrated Kolmogorov (Kolmogorov 1941) spectrum $E(k) \sim k^{-5/3}$. Direct evidence of this classical behaviour has been provided by stirring helium II with counter-rotating discs (Maurer & Tabeling 1998) and by driving helium II along wind tunnels performing measurements behind a grid or a disc (Salort *et al.* 2010). These experimental observations cover a wide temperature range spanning from the critical temperature T_λ down to temperatures where helium II is almost entirely superfluid: all measured spectra perfectly match the Kolmogorov spectrum in the inertial range.

The experimental data have been widely supported by numerical simulations based on distinct models probing the flow at different scales: from the Hall–Vinen–Bekarevic–Khalatnikov model where the smallest resolved scale is larger than ℓ (Roche, Barenghi & Leveque 2009), to the Lagrangian vortex filament method (VFM) discretising quantum vortices in line elements whose lengths are smaller than ℓ but significantly larger than the vortex core (Baggaley, Laurie & Barenghi 2012), to the Gross–Pitaesvkii model, resolving length scales as small as the vortex core (Nore, Abid & Brachet 1997). The contribution of numerical simulations has been fundamental to identifying the mechanisms responsible for the emergence of the Kolmogorov spectrum in QT. The VFM simulations have in fact shown that the Kolmogorov spectrum is directly related to the emergence of quantum vortex bundles, that is regions where quantum vortices are locally aligned and polarised with superfluid vorticity on the lines almost pointing in the same direction.

By coming closer together, these vortex structures can indeed undergo the process of vortex stretching, a mechanism which is known to play an important role in governing the energy cascade (on individual vortices vortex stretching is prevented by the quantisation of the circulation). A similar essential role in shaping the Kolmogorov spectrum is played in CT by elongated regions of large enstrophy (Farge, Pellegrino & Schneider 2001). Quantum vortex bundles can hence effectively be described as classical eddies in ordinary turbulence and this explains why in QT classical features emerge at scales larger than ℓ: to observe a classical behaviour, the dynamics needs to be governed by the collective interaction of many quantum vortices.

The role of vortex bundles in QT has been confirmed in a series of recent numerical studies investigating the scaling of the velocity circulation statistics in the $T = 0$ limit

(Müller *et al.* 2021; Polanco, Müller & Krstulovic 2021). By computing the scaling exponents of circulation moments in the inertial range, in these studies both the self-similar Kolmogorov scalings and intermittency deviations have been observed, the latter closely reflecting the bifractal model for circulation moments recently proposed in CT (Iyer, Sreenivasan & Yeung 2019). Importantly, the polarisations of vortex bundles are found to determine the Kolmogorov self-similar behaviour, while intermittency corrections arise from the non-trivial spatial distribution of vortices. This unveiled universality of the inertial range dynamics reinforces the view that turbulent flows, both quantum and classical, can be described in terms of the collective, mutually interacting dynamics of discrete and thin vortex filaments of fixed circulation. This universal character of the inertial range dynamics is also supported by the observation of classically shaped probability density functions (PDFs) of velocity and acceleration components in QT at scales larger than ℓ (La Mantia *et al.* 2016) employing particle tracking velocimetry of solid hydrogen or deuterium particles (Guo *et al.* 2014): velocity PDFs coincide with Gaussian distributions, while acceleration distributions follow a classical functional form associated with a log-normal distribution of the acceleration magnitude. When probing scales are lowered below ℓ, quantum effects kick in and the velocity PDFs switch dramatically to power laws arising from the interaction of particles with individual vortex lines. It is worth mentioning that when the normal fluid is present ($T > 1.3$ K), similarities between CT and QT may also emerge at small scales where the quantum analogue of the classical dissipation anomaly has been observed (Mäkinen *et al.* 2023; Galantucci *et al.* 2023*b*). To conclude this section, it is of essential importance to emphasise that, when QT is generated thermally, e.g. by placing a heater in a closed channel, several non-classical features emerge. For instance, the vortex line density decays as $L(t) \sim t^{-1}$ (Vinen 1957) and the energy spectra $E(k) \sim k^{-\beta}$ with $\beta > 5/3$ (Gao *et al.* 2017), evidence of a stronger dissipation arising from the mutual friction force which in thermal QT acts at all scales.

3. Conclusions

The recent work by Blaha *et al.* (2025) provides additional evidence that, in flows driven mechanically and at scales larger than the average intervortex spacing, QT displays features akin to CT. This result strongly reinforces the view that the dynamics in the inertial range is universal and, hence, that the backbone of turbulence, both classical and quantum, is constituted by discrete and thin vortex filaments of fixed circulation. This evocative interpretation hints at the idea that the study of QT could lead to a better understanding of CT. Furthermore, the investigation of Blaha *et al.* (2025) shows that vortex trajectories in QT are very similar to the ones observed in CT, suggesting that quantum-like vortex excitations may be observed on classical vortices.

Acknowledgements. The author thanks Mr. P. Stasiak for the graphical abstract.

Declaration of interests. The author reports no conflict of interest.

REFERENCES

BAGGALEY, A.W., LAURIE, J. & BARENGHI, C.F. 2012 Vortex-density fluctuations, energy spectra, and vortical regions in superfluid turbulence. *Phys. Rev. Lett.* **109**, 205304.

BARENGHI, C.F., SKRBEK, L. & SREENIVASAN, K.R. 2023 *Quantum Turbulence*. Cambridge University Press.

BEWLEY, G.P., PAOLETTI, M.S., SREENIVASAN, K.R. & LATHROP, D.P. 2008 Characterization of reconnecting vortices in superfluid helium. *Proc. Nat. Acad. Sci. USA* **105**, 13707.

BLAHA, J., XU, L. & LA MANTIA, M. 2025 Newtonian-Like behaviour of starting vortex flow in superfluid helium at high Reynolds numbers. *J. Fluid Mech.* **1015**, A57.

FARGE, M., PELLEGRINO, G. & SCHNEIDER, K. 2001 Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets. *Phys. Rev. Lett.* **87**, 054501.

GALANTUCCI, L., BAGGALEY, A.W., BARENGHI, C.F. & KRSTULOVIC, G. 2020 Quantum turbulence in superfluid helium: a self-consistent approach. *Eur. Phys. J. Plus* **135**, 547.

GALANTUCCI, L., KRSTULOVIC, G. & BARENGHI, C.F. 2023*a* Friction-enhanced lifetime of bundled quantum vortices. *Phys. Rev. Fluids* **8**, 014702.

GALANTUCCI, L., RICKINSON, E., BAGGALEY, A.W., PARKER, N.G. & BARENGHI, C.F. 2023*b* Dissipation anomaly in a turbulent quantum fluid. *Phys. Rev. Fluids* **8** (3), 034605.

GAO, J., VARGA, E., GUO, W. & VINEN, W.F. 2017 Energy spectrum of thermal counterflow turbulence in superfluid helium-4. *Phys. Rev. B* **96**, 094511.

GUO, W., LA MANTIA, M., LATHROP, D.P. & VAN SCIVER, S.W. 2014 Visualization of two-fluid flows of superfluid helium-4. *Proc. Nat. Acad. Sci. USA* **111** (supplement_1), 4653–4658.

HALL, H.E. & VINEN, W.F. 1956 The rotation of liquid helium ii. ii. the theory of mutual friction in uniformly rotating helium ii. *Proc. R. Soc. London A* **238**, 1213,–215.

HASKELL, B. & MELATOS, A. 2015 Models of pulsar glitches. *Intl. J. Mod. Phys. D* **24**, 530008.

IYER, K.P., SREENIVASAN, K.R. & YEUNG, P. K. 2019 Circulation in high Reynolds number isotropic turbulence is a bifractal. *Phys. Rev. X* **9**, 041006.

KOLMOGOROV, A.N. 1941 The local structure of turbulence in an incompressible viscous fluid for very large Reynolds numbers. *Dokl. Akad. Nauk. SSSR* **30**, 301.

LA MANTIA, M., ŠVANČARA, P., DUDA, D. & SKRBEK, L. 2016 Small-scale universality of particle dynamics in quantum turbulence. *Phys. Rev. B* **94**, 184512.

LANDAU, L.D. 1941 Theory of the superfluidity of helium ii. *J. Phys U.S.S.R.* **5**, 71.

MÄKINEN, J.T., AUTTI, A., HEIKKINEN, P.J., HOSIO, J.J., HÄNNINEN, R., L'VOV, V.S., WALMSLEY, P.M., ZAVJALOV, V.V. & ELTSOV, V.B. 2023 Rotating quantum wave turbulence. *Nat. Phys.* **19**, 898.

MAURER, J. & TABELING, P. 1998 Local investigation of superfluid turbulence. *Europhys. Lett.* **43**, 29.

MÜLLER, N.P., POLANCO, J.I. & KRSTULOVIC, G. 2021 Intermittency of velocity circulation in quantum turbulence. *Phys. Rev. X* **11**, 011053.

NORE, C., ABID, M. & BRACHET, M.E. 1997 Kolmogorov turbulence in low-temperature superflows. *Phys. Rev. Lett.* **78** (20), 3896.

PANICO, R., COMARON, P., MATUSZEWSKI, M., LANOTTE, A.S., TRYPOGEORGOS, D., GIGLI, G., GIORGI, M.D., ARDIZZONE, V., SANVITTO, D. & BALLARINI, D. 2023 Onset of vortex clustering and inverse energy cascade in dissipative quantum fluids. *Nat. Photonics* **17**, 451.

PERETTI, C., VESSAIRE, J., DUROZOY, É. & GIBERT, M. 2023 Direct visualization of the quantum vortex lattice structure, oscillations, and destabilization in rotating ^{4}He. *Sci. Adv.* **9** (30), eadh2899.

POLANCO, J.I., MÜLLER, N.P. & KRSTULOVIC, G. 2021 Vortex clustering, polarisation and circulation intermittency in classical and quantum turbulence. *Nat. Commun.* **12**, 7090.

ROCHE, P.-E., BARENGHI, C.F. & LEVEQUE, E. 2009 Quantum turbulence at finite temperature: the two-fluids cascade. *Europhys. Lett.* **87**, 54006.

SALORT, J., 2010 Turbulent velocity spectra in superfluid flows. *Phys. Fluids* **22**, 125102.

SERAFINI, S., GALANTUCCI, L., ISENI, E., BIENAIMÉ, T., BISSET, R.N., BARENGHI, C.F., DALFOVO, F., LAMPORESI, G. & FERRARI, G. 2017 Vortex reconnections and rebounds in trapped atomic Bose-Einstein condensates. *Phys. Rev. X* **7**, 021031.

SKRBEK, L. & SREENIVASAN, K.R. 2012 Developed quantum turbulence and its decay. *Phys. Fluids* **24**, 011301.

ŠVANČARA, P. & LA MANTIA, M. 2022 Coherent propagation of vortex rings at extremely high Reynolds numbers. *J. Fluid Mech.* **953**, A28.

TOUIL, H., BERTOGLIO, J.-P. & SHAO, L. 2002 The decay of turbulence in a bounded domain. *J. Turbul.* **3**, N49.

VINEN, W.F. 1957 Mutual friction in a heat current in liquid helium ii. iii. theory of mutual friction. *Proc. R. Soc. London A* **242**, 493.

WHITE, A.C., ANDERSON, B.P. & BAGNATO, V.S. 2014 Vortices and turbulence in trapped atomic condensates. *Proc. Nat. Acad. Sci.* **111**, 4719–4726.