J. Fluid Mech. (2025), *vol.* 1018, F1, doi:10.1017/jfm.2025.10382

JFM FOCUS ON FLUIDS

When space gets tight: confinement-driven particulate fingering

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(Received 13 June 2025; revised 13 June 2025; accepted 17 June 2025)

Fingering instabilities readily occur if a less viscous fluid displaces a more viscous fluid in a narrow gap due to the action of destabilising viscous forces. If the fluids are miscible, the instability can be suppressed in the limit of large advection as complicated flow structures are formed across the gap. Using a fluid to displace a monolayer of non-colloidal particles suspended in the same fluid, Luo *et al.* (2025 *J. Fluid Mech.* vol. 1011, A48) suppress the formation of the cross-gap structures and identify a new fingering mechanism which instead relies on long-range dipolar disturbance flows generated by the particle confinement.

Key words: particle/fluid flow, Hele-Shaw flows, fingering instability

1. Fingering instability

When a less viscous fluid displaces a more viscous fluid in a porous medium, its front readily destabilises, developing into complex continually evolving finger-like structures. The mechanism for the onset of this instability is well understood, at least in the simplest scenarios, such as when the two fluids are immiscible and moving in a narrow gap between two parallel walls (i.e. a Hele-Shaw cell). The phenomenon was famously studied by Saffman & Taylor (1958) who showed that in-plane perturbations to the flat interface grow because they select a path of least viscous resistance. This is counteracted by interfacial surface tension that acts to straighten the interface, stabilising it to short-wavelength disturbances, with linear theory predicting a square root scaling with the surface tension for the critical wavelength of the instability. However, long-term growth of the instability tends to be extremely sensitive to the initial conditions of the system, and repeated experiments in the same set-up will typically produce significantly different patterns (Couder 2003).

The large aspect ratios of Hele-Shaw cells mean that the finger dynamics is often successfully modelled by a set of two-dimensional, depth-averaged lubrication equations which assume a parabolic velocity profile in the cell gap, and are mathematically analogous to the Darcy model for flow in a porous medium. Since interfacial growth is typically disordered, the instability is a useful toy model for investigating fundamental ideas of nonlinear dynamics which are hard to test in systems of increased complexity (Lawless, Juel & Pihler-Puzović 2025). The instability also features in many practical applications. Historically, it has been a major obstacle to efficient sweeping in enhanced oil recovery (Woods 2014), and it occurs regularly in hydraulic fracturing (Osiptsov 2017), subsurface sequestration of CO_2 and hydrogen storage (Miocic *et al.* 2023) and when treating groundwater contamination (Clayton 1998) amongst other applications. Unsurprisingly, a significant research effort has been directed towards controlling the onset of instabilities and their long-term evolution, including but not limited to creative ways of injecting the displacing fluid (Li *et al.* 2009), changing the geometry of the confinement (Al-Housseiny, Tsai & Stone 2012) or the constitutive behaviour of flowing fluids (Bischofberger, Ramachandran & Nagel 2014).

2. Miscible displacement

Many engineering systems feature miscible displacement flows, such as when lower viscosity liquid containing proppant is used to displace a more viscous liquid during alternate-slug fracturing; in this scenario, miscible fingering ensures a more uniform placement of particles that maintain the conductive paths through the rock by keeping the fractures open (Osiptsov 2017). The fluid mechanics of miscible fingering is inherently different from that described by Saffman & Taylor (1958), since no interface exists between the invading and displaced fluids. A similar stabilising role to that of surface tension can be played by diffusion, which controls the width of the mixed region, thereby setting the wavelength of the instability, with both predicted to increase with time according to the linear stability analysis (Tan & Homsy 1986). In another regime where advection dominates over diffusion, increasing the viscosity ratio between the invading and displaced fluids (while still maintaining it firmly below one) can also lead to complete suppression of fingering. This surprising phenomenon was first observed by Bischofberger *et al.* (2014) for the case of simple mixtures of water and glycerol, but numerous related studies have since reported a delay in the onset of miscible fingering (Videbæk 2020), including when a suspension of non-colloidal (quasi-neutrally) buoyant particles in oil is displaced by the same oil (Luo, Chen & Lee 2020).

3. Bringing back confinement

The disappearance or delay of fingering during miscible displacement in the advection-dominated regime is counterintuitive, since neither surface tension nor thermal diffusion of colloidal particles are of relevance, and there is no apparent mechanism that can stop the destabilisation induced by the viscosity contrast. However, this simplistic view of the problem undermines the truly three-dimensional nature of the flow, which can lead to the development of complicated structures across the narrow cell gap. For example, in experiments by Luo *et al.* (2020), in-plane fingering disappears when a sharp cross-gap profile develops between the invading oil and the displaced suspension, a fluid with higher effective viscosity. The problem is further complicated by the shear-induced migration of particles that leads to their focusing in the centre of the cell gap, with the blunt tip of the profile becoming rounder in time.

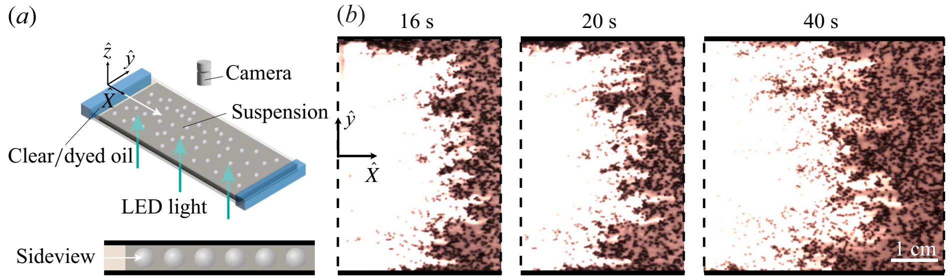

Figure 1. (*a*) Schematic of the experimental set-up: a glass-walled Hele-Shaw cell of gap thickness 0.76 ± 0.03 mm. (*b*) Instantaneous top view images of fingering produced by the clear silicone oil (dynamic viscosity 0.096 Pa s) displacing a suspension of polyethylene particles (diameter 0.625 ± 0.4 mm) in the same oil in the cell from (*a*) (indicted times are from the start of the experiment); particle volume fraction is 25 %, oil is injected at the flow rate of 3.62 ml min^{-1}. Images supplied by R. Luo.

Luo *et al.*'s (2025) recent experiments with particle suspensions examine the same advection-dominated regime, but using a two-dimensional particle suspension. The particles form a monolayer inside a Hele-Shaw cell whose gap thickness is of the order of the particle diameter (figure 1*a*). The new set-up is an ingenious way to study a displacement flow prone to fingering while preventing the formation of cross-gap flow structures. In contrast to all previous work in the advection-dominated regime, Luo *et al.* (2025) observe fingering regardless of the (effective) viscosity ratio. Moreover, their systematic experiments performed by varying volume fractions of particles reveal the emergence of an average fingering wavelength which evolves over time (figure 1*b*). However, the wavelength that emerges is not set by diffusion as is the case for the miscible fingers studied by Tan & Homsy (1986).

The key ingredient in the observed behaviour is the hydrodynamic interaction between particles, which results in a collective dynamics akin to that observed in other two-dimensional suspensions (Desreumaux *et al.* 2013) and clustering of magnetically driven micro-rollers (Driscoll *et al.* 2017). The confinement of particles slows them down relative to the local oil flow, which in turn induces a dipolar disturbance to the suspending oil. The slowly decaying disturbances lead to the development of a rarefaction region ahead of the clear oil that displaces the suspension. In this rarefaction region, which expands over time, the number density of the particles gradually increases from zero to the number density of the particles in the suspension. By combining experiments and linear stability analysis of a two-dimensional coarse-grained model, Luo *et al.* (2025) show that the length scale of the rarefaction region sets the average length scale of the fingering instability. Furthermore, they demonstrate that the coarsening of fingers is independent of the initial particle density, which only controls the time scales of the instability.

4. Future

In their work, Luo *et al.* (2025) have identified a new destabilisation mechanism that leads to miscible fingering in suspensions. This physical effect weakens over time, as often seen during various fingering phenomena, such as the aforementioned diffusion-controlled fingering studied by Tan & Homsy (1986), or when immiscible flow is decelerated below an inflating elastic sheet making viscous forces less destabilising (Pihler-Puzović *et al.* 2018).

It is yet to be quantified how and where the new mechanism becomes irrelevant and if instead the flow transitions to the particulate fingering explored by Luo *et al.* (2020), which bears resemblance to other miscible fingering phenomena in the limit of strong advection. This requires more experiments supported by mathematical modelling, in which the gap thickness and the particle density are systematically varied. Another interesting limit is the case in which the number of suspended particles approaches the maximum packing volume fraction within the monolayer. In this limit, the instability discussed here is likely more similar to the fingering explored by Sandnes *et al.* (2011), in which the frictional interactions between particles are significant. Understanding the impact of sidewalls, non-uniform particle distributions, or soft response of particles are just some of the many outstanding questions for practical implications of the phenomenon unravelled by Luo *et al.* (2025).

Finally, the set-up of Luo *et al.* (2025) offers exciting possibilities for further fundamental studies in fingering. For example, Chevalier, Lindner & Clément (2007) explored a subcritical transition to disorder of a steadily advancing symmetric air finger displacing a particulate suspension in a Hele-Shaw cell, in which the volume fraction of particles was systematically increased. Using the two-dimensional suspensions instead could provide another means of changing finite-amplitude perturbations to the advancing finger.

Acknowledgements. D.P.P. is grateful to R. Luo and S. Lee for reading earlier versions of this article and supplying the images for it. D.P.P. would also like to thank F. Box and A. Juel for their valuable feedback on the article.

Declaration of interests. The author reports no conflict of interest.

REFERENCES

AL-HOUSSEINY, T.T., TSAI, P.A. & STONE, H.A. 2012 Control of interfacial instabilities using flow geometry. *Nat. Phys.* **8** (10), 747–750.

BISCHOFBERGER, I., RAMACHANDRAN, R. & NAGEL, S.R. 2014 Fingering versus stability in the limit of zero interfacial tension. *Nat. Commun.* **5** (1), 5265.

CHEVALIER, C., LINDNER, A. & CLÉMENT, E. 2007 Destabilization of a Saffman–Taylor fingerlike pattern in a granular suspension. *Phys. Rev. Lett.* **99** (17), 174501.

CLAYTON, W.S. 1998 A field and laboratory investigation of air fingering during air sparging. *Ground Water Monit. Remediat.* **18** (3), 134–145.

COUDER, Y. 2003 Viscous fingering as an archetype for growth patterns. In *Perspectives in Fluid Dynamics: A Collective Introduction to Current Research* (ed. G.K. Batchelor, H.K. Moffatt & M.G. Worster), chap. 2, pp. 53–98. Cambridge University Press.

DESREUMAUX, N., CAUSSIN, J.-B., JEANNERET, R., LAUGA, E. & BARTOLO, D. 2013 Hydrodynamic fluctuations in confined particle-laden fluids. *Phys. Rev. Lett.* **111** (11), 118301.

DRISCOLL, M., DELMOTTE, B., YOUSSEF, M., SACANNA, S., DONEV, A. & CHAIKIN, P. 2017 Unstable fronts and motile structures formed by microrollers. *Nat. Phys.* **13** (4), 375–379.

LAWLESS, J., JUEL, A. & PIHLER-PUZOVIĆ, D. 2025 Nonlinear dynamics of viscous fingering. *Phys. D: Nonlinear Phenom.* **476**, 134631.

LI, S., LOWENGRUB, J.S., FONTANA, J. & PALFFY-MUHORAY, P. 2009 Control of viscous fingering patterns in a radial Hele–Shaw cell. *Phys. Rev. Lett.* **102** (17), 174501.

LUO, R., CHEN, Y. & LEE, S. 2020 Particle-induced miscible fingering: continuum limit. *Phys. Rev. Fluids* **5** (9), 094301.

LUO, R., MARSHALL, M., HE, Z., WANG, L. & LEE, S. 2025 Confiment-induced spreading and fingering of suspensions. *J. Fluid Mech.* **1011**, A48.

MIOCIC, J., HEINEMANN, N., EDLMANN, K., SCAFIDI, J., MOLAEI, F. & ALCALDE, J. 2023 Underground hydrogen storage: a review. *Geol. Soc. Spec. Publi.* **528** (1), 73–86.

OSIPTSOV, A.A. 2017 Fluid mechanics of hydraulic fracturing: a review. *J. Petrol. Sci. Engng* **156**, 513–535.

PIHLER-PUZOVIĆ, D., PENG, G.G., LISTER, J.R., HEIL, M. & JUEL, A. 2018 Viscous fingering in a radial elastic-walled Hele–shaw cell. *J. Fluid Mech.* **849**, 163–191.

SAFFMAN, P.G. & TAYLOR, G.I. 1958 The penetration of a fluid into a porous medium or Hele–haw cell containing a more viscous liquid. *Proc. R. Soc. (London) A.* **245**, 312–329.

SANDNES, B., FLEKKØY, E.G., KNUDSEN, H.A., MÅLØY, K.J. & SEE, H. 2011 Patterns and flow in frictional fluid dynamics. *Nat. Commun.* **2** (1), 288.

TAN, C.T. & HOMSY, G.M. 1986 Stability of miscible displacements in porous media: rectilinear flow. *Phys. Fluids* **29** (11), 3549–3556.

VIDEBÆK, T.E. 2020 Delayed onset and the transition to late time growth in viscous fingering. *Phys. Rev. Fluids* **5** (12), 123901.

WOODS, A.W. 2014 *Flow in Porous Rocks: Energy and Environmental Applications*. Cambridge University Press.