

CORRIGENDUM: “ON THE REPRESENTABILITY OF ACTIONS OF LEIBNIZ ALGEBRAS AND POISSON ALGEBRAS”

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In the article [2], the representability of actions of the categories $\mathbf{LeibAlg}_{\mathbb{F}}$ of Leibniz algebras and $\mathbf{PoisAlg}_{\mathbb{F}}$ of Poisson algebras was studied. It was proved that $\mathbf{LeibAlg}_{\mathbb{F}}$ is a weakly action representable category, with a weak actor of a Leibniz algebra \mathfrak{g} being the Leibniz algebra $\mathbf{Bider}(\mathfrak{g})$ of biderivations of \mathfrak{g} .

We recently discovered that the proofs (ii) \Rightarrow (i) of Theorem 5.9 and Theorem 5.10 are not correct. More in detail, the composition $i^* \circ \mu \circ \tau^{-1}$ gives rise to a monomorphism of functors:

$$\mathrm{Hom}_{\mathbf{NAlg}_{\mathbb{F}}^2}(U(-), [V]) \hookrightarrow \mathrm{Hom}_{\mathbf{NAlg}_{\mathbb{F}}^2}(U(-), M),$$



where $U: \mathbf{PoisAlg}_{\mathbb{F}} \rightarrow \mathbf{NAlg}_{\mathbb{F}}^2$ denotes the forgetful functor. Thus, the *Yoneda Lemma* cannot be applied and we cannot conclude that $[V]$ is a Poisson algebra.

As a consequence, we cannot conclude that the category $\mathbf{CPoisAlg}_{\mathbb{F}}$ of commutative Poisson algebra is not weakly action representable and the open problem at the end of the manuscript is not correctly stated. We have attempted to resolve this issue but unfortunately we have not been successful.

However, the other statements of Section 5 of the manuscript are not affected by this mistake and the open problem can be rephrased as follows.

Open Problem

Our investigation does not clarify whether the categories $\mathbf{PoisAlg}_{\mathbb{F}}$ and $\mathbf{CPoisAlg}_{\mathbb{F}}$ are weakly action representable or not. Nevertheless, the explicit construction of the universal strict general actor $\text{USGA}(V)$ provides an operational tool to represent actions on/split extensions by a (commutative) Poisson algebra V . Notice that this construction was generalized for a general variety of non-associative algebras over a field in [1], where the authors proved that the category of commutative associative algebras is weakly action representable, even though the USGA is not an object of it (see [1, Theorem 2.11]).

We further observe that the first examples of action accessible varieties of non-associative algebras that fail to be weakly action representable were found in [3], where it was proved that the categories of n -solvable Lie algebras ($n \geq 2$) and k -nilpotent Lie algebras ($k \geq 3$) do not satisfy weak action representability.

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