The Bulletin of Symbolic Logic
Volume 31, Number 2, June 2025

THESIS ABSTRACTS

The Association for Symbolic Logic publishes abstracts of recent PhD theses in logic. The aim of this activity is to publish abstracts for the majority of recent PhD theses in logic world wide and submitted abstracts will therefore only be edited to ensure that they fall within the general area of logic and are appropriate in terms of length and content. This section willprovide a permanent publicly accessible overview of theses in logic and thus make up for the lack of central repository for the theses themselves. The Thesis Abstracts Section is edited by Sandra Müller. Any abstract should formally be submitted by the thesis advisor though it is expected to usually be prepared by the candidate. For detailed instructions for preparation and submission, including the required TeX template, please consult the link below. https://aslonline.org/journals/the-bulletin-of-symbolic-logic/logic-thesis-abstracts-in-the-bulletin-of-symbolic-logic/.

Yudai Suzuki. ***Studies on Partial Impredicativity in Formal Systems of Arithmetic and Computability Theory***. Tohoku University. 6-3, Aza-Aoba, Aramaki, Aoba-ku, Sndai, Miyagi, Japan. 2024. Supervised by Keita Yokoyama. MSC: 03F35, 03D30. Keywords: reverse mathematics, Weihrauch degrees, predicativity

Abstract

In this thesis, we study the complexity of theorems that may be considered partially impredicative from the point of view of reverse mathematics and Weihrauch degrees.

From the perspective of reverse mathematics and ordinal analysis, the axiomatic system ATR_0 is known as the limit of predicativity, and Π^1_1-CA_0 is known as an impredicative system. In this thesis, we study the complexity of some theorems that are stronger than ATR_0 and weaker than Π^1_1-CA_0 from the point of view of reverse mathematics and Weihrauch degrees.

In Chapter 3, we study some problems related to Knaster–Tarski's theorem. Knaster–Tarski's theorem states that any monotone operator on 2^ω has a least fixed point. Avigad introduced a weaker variant, FP, which asserts the existence of a fixed point instead of the least fixed point, and proved that FP for arithmetical operators is equivalent to ATR_0 over RCA_0. In this thesis, we show that FP for Σ^0_2-operators is strictly stronger than ATR_2, a Weihrauch degree corresponding to ATR_0, in terms of Weihrauch reduction. In addition, we study the bottom-up proof of Knaster–Tarski's theorem. It is known that the least fixed point of a monotone operator is given by the ω_1-times iteration of the operator at the empty set. This implies that any monotone operator involves a hierarchy formed by the iterative applications of the operator, starting with the empty set and reaching the least fixed point. We prove that although the existence of a hierarchy is equivalent to ATR_0 over ACA_0, it is stronger than $\mathsf{C}_{\omega^\omega}$ in the terms of Weirhauch reduction.

In Chapter 5, we study the relative leftmost path principle in Weihrauch degrees. This principle was introduced by Towsner to study partial impredicativity in reverse mathematics. He gave a hierarchy between ATR_0 and Π^1_1-CA_0 by this principle. We show that this principle also makes a hierarchy between ATR_2 and $\mathsf{C}_{\omega^\omega}$ in Weihrauch degrees. We also show that the

1079-8986/25/3102-0005
DOI :10.1017/bsl.2024.35

relative leftmost path principle is, not the same as, but very close to a variant of β-model reflection.

In Chapter 6, we introduce a hierarchy dividing $\{\sigma \in \Pi^1_2 : \Pi^1_1\text{-}\mathsf{CA}_0 \vdash \sigma\}$. Then, we give some characterizations of this hierarchy using some principles equivalent to $\Pi^1_1\text{-}\mathsf{CA}_0$: leftmost path principle, Ramsey's theorem for Σ^0_n classes of $[\mathbb{N}]^{\mathbb{N}}$ and the determinacy of Gale–Stewart game for $(\Sigma^0_1)_n$ classes. As an application, our hierarchy explicitly shows that the number of applications of the hyperjump operator needed to prove Σ^0_n Ramsey's theorem or $(\Sigma^0_1)_n$ determinacy increases when the subscript n increases.

Abstract prepared by Yudai Suzuki.
E-mail: yudai.suzuki.research@gmail.com.

Stephen Mackereth. ***Logic, Arithmetic, and Definitions***. University of Pittsburgh 2024. Supervised by Anil Gupta. MSC: 00A30, 03B30, 03F10, 03F30, 03F35, 03F50. Keywords: Frege, logicism, neologicism, abstractionism, Hume's Principle, conservativeness, definitions, Gödel, Dialectica translation, Hilbert's Program, constructivism.

Abstract

Arithmetic and logic seem to enjoy an especially close relationship. Frege once wrote that arithmetic is reason's nearest kin. To deny any of the basic laws of arithmetic seems tantamount to denying a basic law of logic. My dissertation is concerned with two great attempts to make something more of this informal idea. In one direction, Frege tried to reduce arithmetic to nothing but quantificational logic and definitions. *Neologicists* continue to follow in Frege's footsteps, pursuing a version of this program today. In the other direction, Gödel tried to reduce certain applications of quantificational logic to nothing but arithmetic and definitions, by means of his *Dialectica translation*.

In the first half of my dissertation, I prove new theorems (with Jeremy Avigad) that shed a surprising light on the prospects for neologicism. An important objection against neologicism is that it makes use of allegedly stipulative definitions that are not *conservative* over pure logic, i.e., definitions that yield new consequences expressible in old vocabulary. This violates a basic requirement on stipulative definitions. I argue that by passing to a richer logical and definitional framework, it is possible to overcome the conservativeness objection. However, there is a subtlety: the strategy succeeds only if conservativeness is understood semantically rather than deductively. This suggests that the viability of neologicism is highly sensitive to the way in which epistemic commitments are represented in formal theories.

In the second half of my dissertation, I argue that Gödel's Dialectica translation succeeds in assigning a constructive meaning to quantificational theories of arithmetic. Virtually all commentators have objected that Gödel's translation makes use of definitions which presuppose the very quantificational logic that Gödel was trying to eliminate. This would render the translation philosophically circular. Gödel was adamant that there was no circularity here. He attempted to explain the matter in a page-long footnote, which, however, no one has been able to understand. I vindicate Gödel, showing that there is no circularity and answering a longstanding exegetical question in Gödel scholarship.

Abstract prepared by Stephen Mackereth
E-mail: sgmackereth@gmail.com
URL: http://d-scholarship.pitt.edu/id/eprint/46578